Graphing Equations

Britannica® Mathematics in Context

Algebra

TEACHER'S GUIDE

ENCYCLOPÆDIA Britannica®

Mathematics in Context is a comprehensive curriculum for the middle grades. It was developed in 1991 through 1997 in collaboration with the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht, The Netherlands, with the support of the National Science Foundation Grant No. 9054928.

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Opinions expressed are those of the authors and not necessarily those of the Foundation.

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The Teacher's Guide for this unit was prepared by David C. Webb, Jean Krusi, Monica Wijers, and Dédé de Haan.

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Development 1991–1997

The initial version of *Graphing Equations* was developed by Martin Kindt and Monica Wijers. It was adapted for use in American schools by Mary S. Spence, Lora J. Brinker, Margie A. Pligge, and Jack Burrill.

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Graphing Equations and the NCTM Principles and Standards for School Mathematics for Grades 6–8

The process standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation are addressed across all *Mathematics in Context* units.

In addition, this unit specifically addresses the following PSSM content standards and expectations:

Number and Operations

In grades 6–8 all students should:

- develop meaning for integers and represent and compare quantities with them;
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers; and
- understand and use the inverse relationships of addition and subtraction, multiplication and division, to solve problems.

Algebra

In grades 6–8 all students should:

- relate and compare different forms of representation for a relationship;
- develop an initial conceptual understanding of different uses of variables;
- explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;
- use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- recognize and generate equivalent forms for simple algebraic expressions and solve linear equations;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations; and
- use graphs to analyze the nature of changes in quantities in linear relationships.

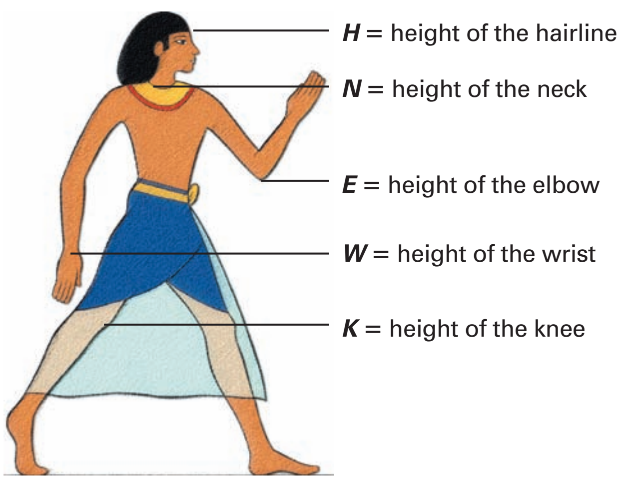
Math in the Unit

Prior Knowledge

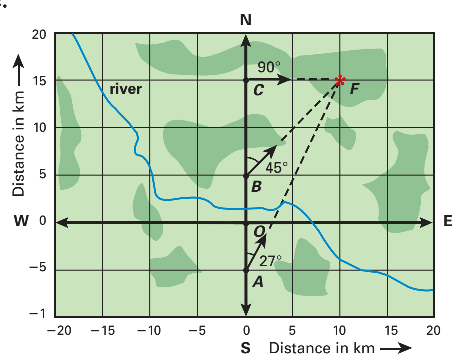
This unit assumes students can do the following:

- add, subtract, multiply, and divide rational numbers in all representations (decimals, percentages, fractions);
- use metric measurements;
- graph lines (as introduced and used in the units *Expressions and Formulas* and *Ups and Downs*);
- read and use compass directions and measure angles with a compass card or protractor;
- read and use coordinates on a coordinate grid (developed in *Expressions and Formulas, Operations,* and *Ups and Downs*);
- understand inequality notation (introduced in the unit *Operations*);
- read simple formulas relating two variables (developed in *Building Formulas* and *Ups and Downs*); and
- perform simple operations with negative numbers (introduced in the unit *Operations*).

This unit should be taught after the units *Expressions and Formulas, Operations, Building Formulas,* and *Ups and Downs.*



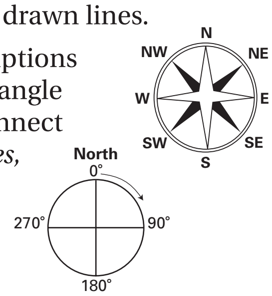
In *Graphing Equations,* students move from locating points using compass directions and bearings to locating points on a coordinate system in the context of a forest fire.



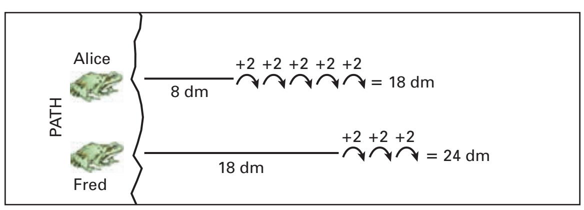
They continuously formalize their knowledge, building on the introduction from the unit *Operations* and adopt conventional formal vocabulary and notation, such as *origin*, *quadrant*, and *x*-axis, as well as the *ordered pairs notation (x, y).*

Starting from steps along a line, students investigate the role of the slope as a rule for finding coordinates. The use of the *y*-intercept as a reference point for graphing linear functions, is formally introduced in this unit. Students draw lines for given equations and write equations for drawn lines.

In using different descriptions of directions and using angle measurements, they connect to *Figuring All the Angles, Triangles and Beyond,* and *It's All the Same.*



Students study the concept of slope and *y*-intercept and write equations for straight lines. Visualizing frogs jumping toward or away from a path helps students develop formal algebraic methods for solving linear equations. By simultaneously changing the diagrams and the equations the diagram visualizes to solve a problem, students learn to understand and use a formal way of solving equations.



Students learn to write down the operations they perform to keep track of the steps they take in solving the equations. They also describe and graph problem situations, which they solve by locating the point of intersection.

$15 + 8x = 37 - 3x$	
$15 + 11x = 37$	Add $3x$ to both sides.
$11x = 22$	Subtract 15 from both sides.
$x = 2$	Divide both sides by 11.

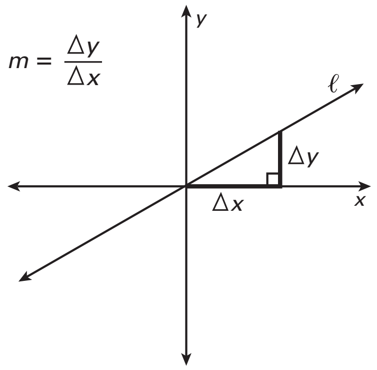
Students combine the graphic method to find a point of intersection with the use of equations.

By linking the lines in the graph to their equations using arrows, the method for solving frog problems is related to finding the point of intersection of two lines.

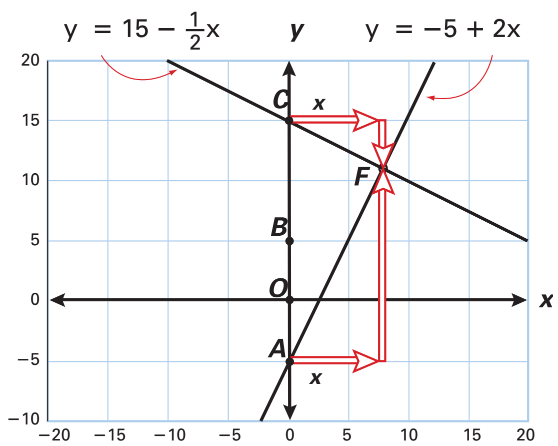
They connect the graphic and algebraic method explicitly for a deeper understanding of the two. Students also explore the relationship between parallel lines and graphs of lines without intersection points.

When students have finished the unit they will:

- understand the use of compass directions and angle measurements by:
  - revisiting the navigation and angle measurement concepts introduced in previous geometry units; and
  - describing directions using ordered number pairs.



- Understand the relationship between equation and graph of a linear function by:
  - studying and using the concept of slope at a more formal level;
  - understanding the relationship between slope and tangent ratio of a line;
  - graphing points and lines in a coordinate system;
  - finding the y-intercept using the equation of a line or the graph and understanding its meaning;
  - finding and using equations of a straight line;
  - using inequalities to describe a region;
  - finding the intersection point of two straight lines by reading from the graph and checking as well as by solving a linear equation; and
  - understanding similarities between graphic and algebraic strategies.
- Solve single variable linear equations by:
  - writing and solving linear equations.



Algebra Strand: An Overview

Mathematical Content

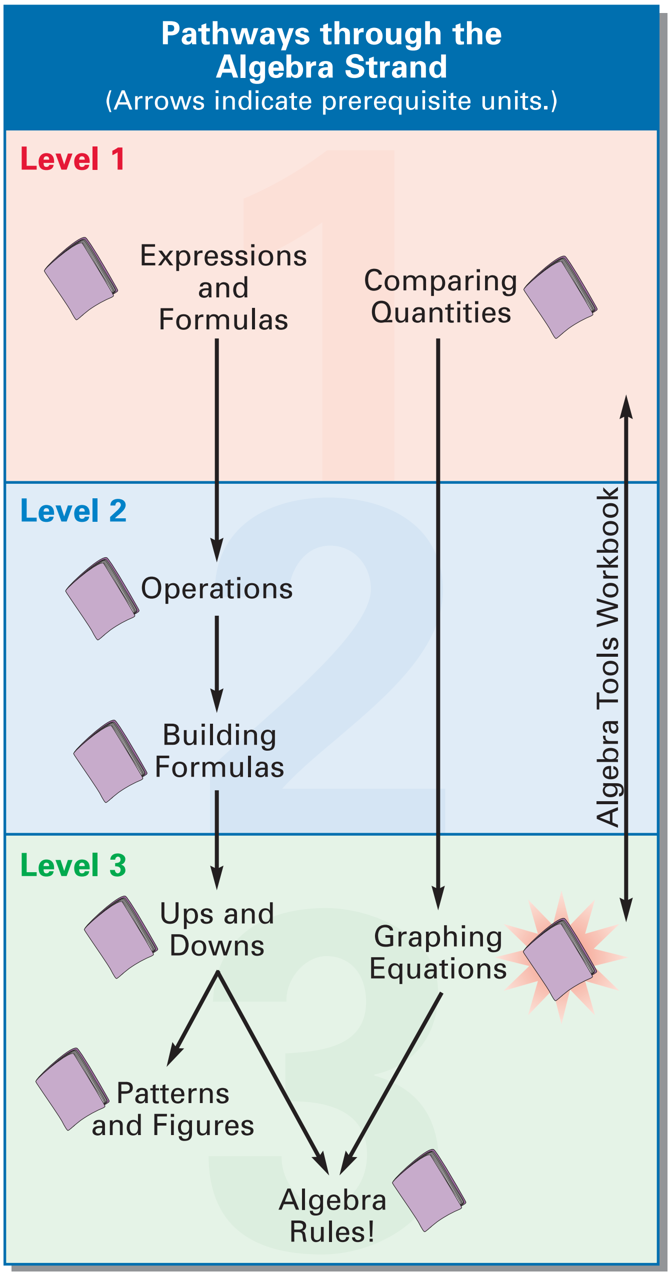
The Algebra strand in *Mathematics in Context* emphasizes algebra as a language used to study relationships among quantities. Students learn to describe these relationships with a variety of representations and to make connections among these representations. The goal is for students to understand the use of algebra as a tool to solve problems that arise in the real world or in the world of mathematics, where symbolic representations can be temporarily freed of meaning to bring a deeper understanding of the problem. Students move from preformal to formal strategies to solve problems, learning to make reasonable choices about which algebraic representation, if any, to use. The goals of the units within the algebra strand are aligned with NCTM's *Principles and Standards for School Mathematics.*

Algebra Tools and Other Resources

The *Algebra Tools* Workbook provides materials for additional practice and further exploration of algebraic concepts that can be used in conjunction with units in the Algebra Strand or independently from individual units. The use of a graphing calculator is optional in the student books. The Teacher's Guides provide additional questions if graphing calculators are used.

Organization of the Algebra Strand

The theme of change and relationships encompasses every unit in the Algebra strand. The strand is organized into three substrands: Patterns and Regularities, Restrictions, and Graphing. Note that units within a substrand are also connected to units in other substrands.

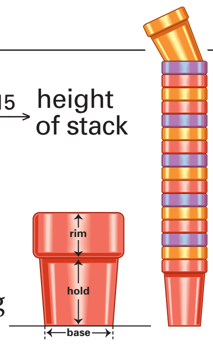


Patterns and Regularities

In the Patterns and Regularities substrand, students explore and represent patterns to develop an understanding of formulas, equations, and expressions. The first unit, *Expressions and Formulas*, uses arrow language and arithmetic trees to represent situations. With these tools, students create and use word formulas that are the precursors to algebraic equations. The problem below shows how students use arrow language to write and solve equations with a single unknown.

The students use an arrow string to find the height of a stack of cups.

number of cups $\xrightarrow{-1}$ ___ $\xrightarrow{\times 3}$ ___ $\xrightarrow{+15}$ height of stack



a. How tall is a stack of ten of these cups?

b. Explain what each of the numbers in the arrow string represents.

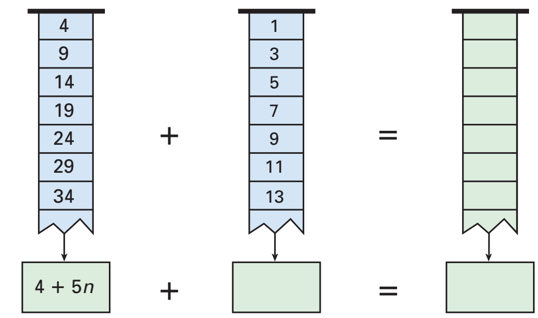
c. These cups need to be stored in a space 50 cm high. How many of these cups can be placed in a stack? Explain how you found your answer.

As problems and calculations become more complicated, students adapt arrow language to include multiplication and division. When dealing with all four arithmetic operations, students learn about the order of operations and use another new tool—arithmetic trees—to help them organize their work and prioritize their calculations. Finally, students begin to generalize their calculations for specific problems using word formulas.

saddle height (in cm) = inseam (in cm) × 1.08
frame height (in cm) = inseam (in cm) × 0.66 + 2

In *Building Formulas*, students explore direct and recursive formulas (formulas in which the current term is used to calculate the next term) to describe patterns. By looking at the repetition of a basic pattern, students are informally introduced to the distributive property. In *Patterns and Figures*, students continue to use and formalize the ideas of direct and recursive formulas and work formally with algebraic expressions, such as $2(n + 1)$.

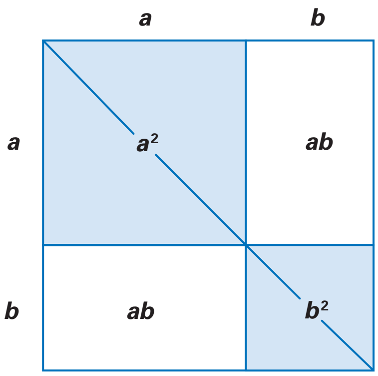
In a recursive (or NEXT-CURRENT) formula, the next number or term in a sequence is found by performing an operation on the current term according to a formula. For many of the sequences in this unit, the next term is a result of adding or subtracting a fixed number from the current term of the sequence. Operations with linear expressions are connected to "Number Strips," or arithmetic sequences.



Students learn that they can combine sequences by addition and subtraction. In *Patterns and Figures*, students also encounter or revisit other mathematical topics such as rectangular and triangular numbers. This unit broadens their mathematical experience and makes connections between algebra and geometry.

Overview

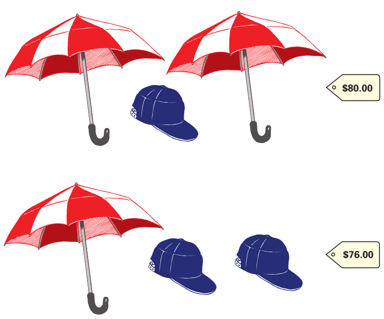
In the unit *Graphing Equations*, linear equations are solved in an informal and preformal way. The last unit, *Algebra Rules!*, integrates and formalizes the content of algebra substrands. In this unit, a variety of methods to solve linear equations is used in a formal way. Connections to other strands are also formalized. For example, area models of algebraic expressions are used to highlight relationships between symbolic representations and the geometry and measurement strands. In *Algebra Rules!*, students also work with quadratic expressions.



The Patterns and Regularities substrand includes a unit that is closely connected to the Number strand, *Operations*. In this unit, students build on their informal understanding of positive and negative numbers and use these numbers in addition, subtraction, and multiplication. Division of negative numbers is addressed in *Revisiting Numbers* and in *Algebra Rules!*

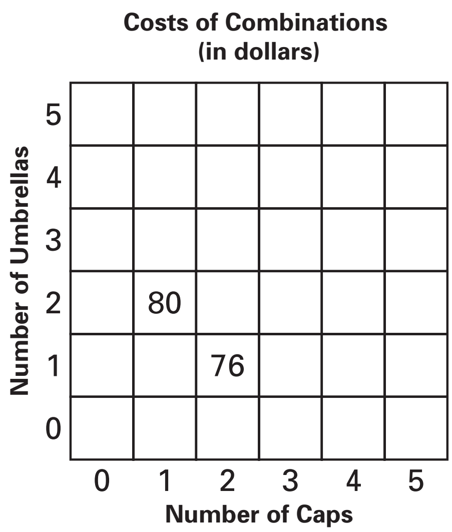
Restrictions

In the Restrictions substrand, the range of possible solutions to the problems is restricted because the mathematical descriptions of the problem contexts require at least two equations. In *Comparing Quantities*, students explore informal methods for solving systems of equations through nonroutine, yet realistic, problem situations such as running a school store, renting canoes, and ordering in a restaurant.



Within such contexts as bartering, students are introduced to the concept of substitution (exchange) and are encouraged to use symbols to represent problem scenarios. Adding and subtracting relationships graphically and multiplying the values of a graph by a number help students develop a sense of operations with expressions.

To solve problems about the combined costs of varying quantities of such items as pencils and erasers, students use charts to identify possible combinations. They also identify and use the number patterns in these charts to solve problems.



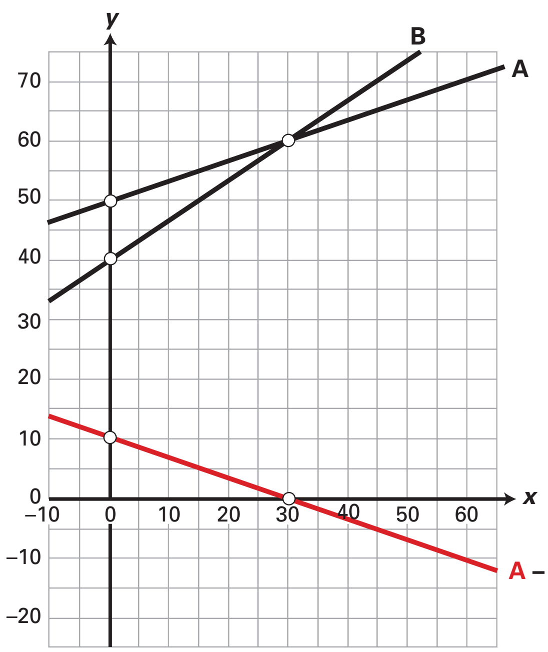
Students' work with problems involving combinations of items is extended as they explore problems about shopping. Given two "picture equations" of different quantities of two items and their combined price, students find the price of a single item. Next, they informally solve problems involving three equations and three variables within the context of a restaurant and the food ordered by people at different tables.

This context also informally introduces matrices. At the end of the unit, students revisit these problem scenarios more formally as they use variables and formal equations to represent and solve problems.

ORDER	TACO	SALAD	DRINK	TOTAL
1	2	4	—	$10
2	1	2	3	$8
3	3	—	3	$9
4	1	2	—	
5	1	—	1	
6	2	2	1	
7	4	2	3	
8				
9				
10				

In *Graphing Equations*, students move from locating points using compass directions and bearings to using graphs and algebraic manipulation to find the point of intersection of two lines.

Students may use graphing calculators to support their work as they move from studying slope to using slope to write equations for lines. Visualizing frogs jumping toward or away from a path helps students develop formal algebraic methods for solving a system of linear equations. In *Algebra Rules!*, the relationship between the point of intersection of two lines (A and B) and the x-intercept of the difference between those two lines (A – B) is explored. Students also find that parallel lines relate to a system of equations that have no solution.

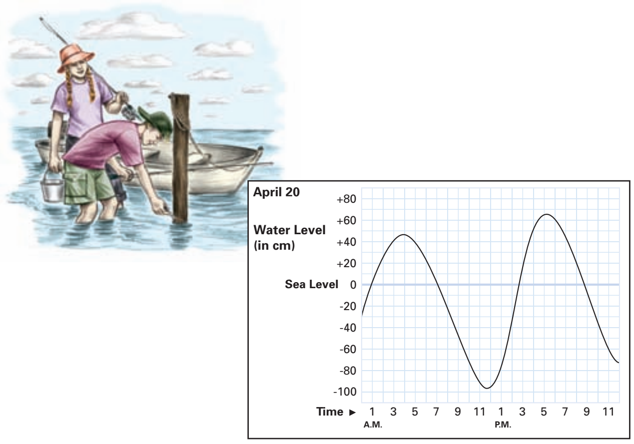


Graphing

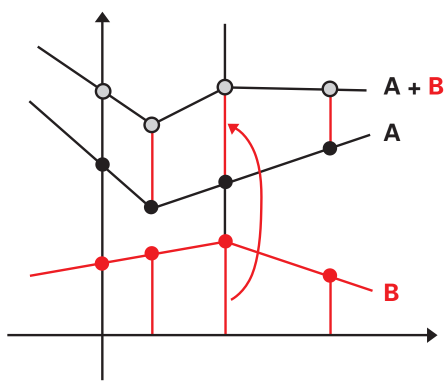
The Graphing substrand, which builds on students' experience with graphs in previous number and statistics units, begins with *Expressions and Formulas* where students relate formulas to graphs and read information from a graph.

Operations, which is in the Patterns and Regularities substrand, is also related to the Graphing substrand since it formally introduces the coordinate system.

In *Ups and Downs*, students use equations and graphs to investigate properties of graphs corresponding to a variety of relationships: linear, quadratic, and exponential growth as well as graphs that are periodic.



In *Graphing Equations*, students explore the equation of a line in slope and y-intercept form. They continuously formalize their knowledge and adopt conventional formal vocabulary and notation, such as origin, quadrant, and x-axis, as well as the ordered pairs notation (x, y). In this unit, students use the slope-intercept form of the equation of a line, $y = mx + b$. Students may use graphing calculators to support their work as they move from studying slope to using slope to write equations for lines. Students should now be able to recognize linearity from a graph, a table, and a formula and know the connections between those representations. In the last unit in the Algebra strand, *Algebra Rules!*, these concepts are formalized and the x-intercept is introduced. Adding and subtracting relationships graphically and multiplying the values of a graph by a number help students develop a sense of operations with expressions.



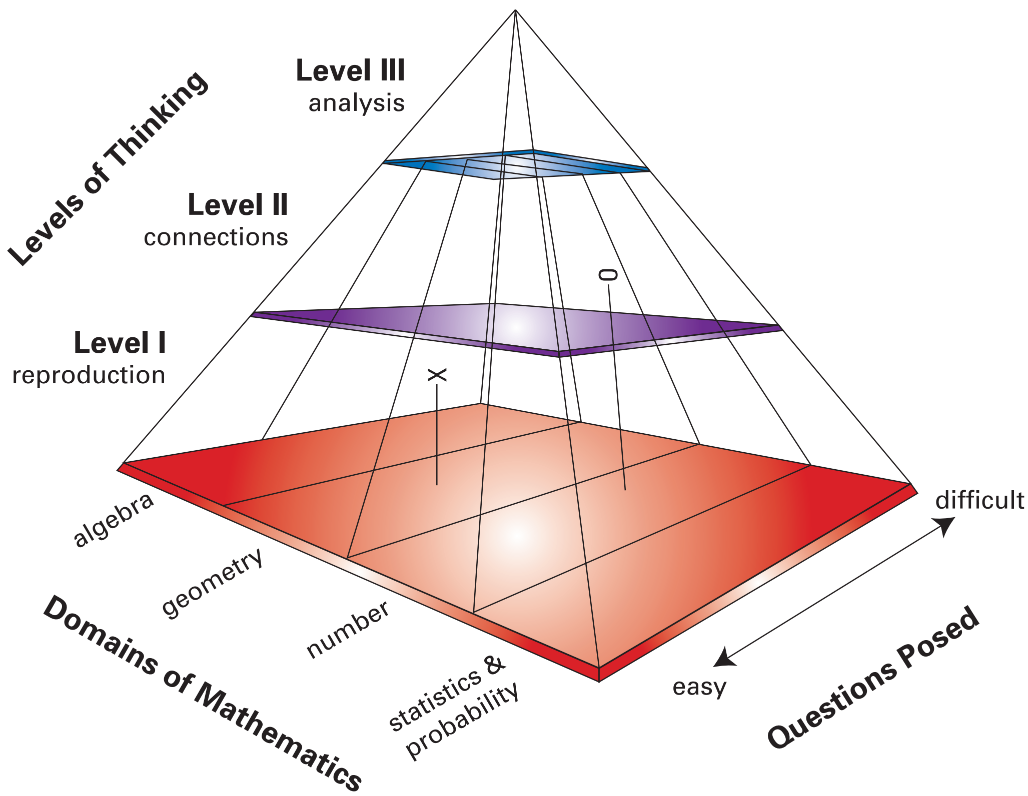
Student Assessment in Mathematics in Context

As recommended by the NCTM *Principles and Standards for School Mathematics* and research on student learning, classroom assessment should be based on evidence drawn from several sources. An assessment plan for a *Mathematics in Context* unit may draw from the following overlapping sources:

- **observation—As students work individually or in groups, watch for evidence of their understanding of the mathematics.**
- **interactive responses—Listen closely to how students respond to your questions and to the responses of other students.**
- **products—Look for clarity and quality of thought in students' solutions to problems completed in class, homework, extensions, projects, quizzes, and tests.**

Assessment Pyramid

When designing a comprehensive assessment program, the assessment tasks used should be distributed across the following three dimensions: mathematics content, levels of reasoning, and difficulty level. The Assessment Pyramid, based on Jan de Lange's theory of assessment, is a model used to suggest how items should be distributed across these three dimensions. Over time, assessment questions should "fill" the pyramid.



Levels of Reasoning

Level I questions typically address:

- recall of facts and definitions and
- use of technical skills, tools, and standard algorithms.

As shown in the pyramid, Level I questions are not necessarily easy. For example, Level I questions may involve complicated computation problems. In general, Level I questions assess basic knowledge and procedures that may have been emphasized during instruction. The format for this type of question is usually short answer, fill-in, or multiple choice. On a quiz or test, Level I questions closely resemble questions that are regularly found in a given unit substituted with different numbers and/or contexts.

Level II questions require students to:

- integrate information;
- decide which mathematical models or tools to use for a given situation; and
- solve unfamiliar problems in a context, based on the mathematical content of the unit.

Level II questions are typically written to elicit short or extended responses. Students choose their own strategies, use a variety of mathematical models, and explain how they solved a problem.

Level III questions require students to:

- make their own assumptions to solve open-ended problems;
- analyze, interpret, synthesize, reflect; and
- develop one's own strategies or mathematical models.

Level III questions are always open-ended problems. Often, more than one answer is possible, and there is a wide variation in reasoning and explanations. There are limitations to the type of Level III problems that students can be reasonably expected to respond to on time-restricted tests.

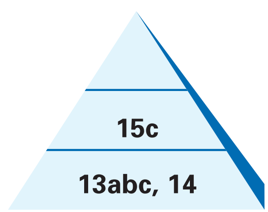
The instructional decisions a teacher makes as he or she progresses through a unit may influence the level of reasoning required to solve problems. If a method of problem solving required to solve a Level III problem is repeatedly emphasized during instruction, the level of reasoning required to solve a Level II or III problem may be reduced to recall knowledge, or Level I reasoning. A student who does not master a specific algorithm during a unit but solves a problem correctly using his or her own invented strategy, may demonstrate higher-level reasoning than a student who memorizes and applies an algorithm.

The "volume" represented by each level of the Assessment Pyramid serves as a guideline for the distribution of problems and use of score points over the three reasoning levels.

These assessment design principles are used throughout *Mathematics in Context*. The Goals and Assessment charts that highlight ongoing assessment opportunities—on pages xvi and xvii of each Teacher's Guide—are organized according to levels of reasoning.

In the Lesson Notes section of the Teacher's Guide, ongoing assessment opportunities are also shown in the Assessment Pyramid icon located at the bottom of the Notes column.

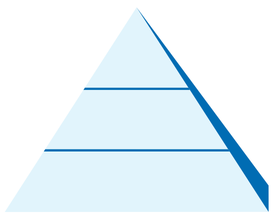
Assessment Pyramid



Understand slope;
Identify direction pairs.

Goals and Assessment

In the *Mathematics in Context* curriculum, unit goals, organized according to levels of reasoning described in the Assessment Pyramid on page xiv, relate to the strand goals and the NCTM Principles and Standards for School Mathematics. The *Mathematics in Context* curriculum is designed to help students demonstrate their understanding of mathematics in each of the categories listed below. Ongoing assessment opportunities are also indicated on their respective pages throughout the Teacher's Guide by an Assessment Pyramid icon.



It is important to note that the attainment of goals in one category is not a prerequisite to the attainment of those in another category. In fact, students should progress simultaneously toward several goals in different categories. The Goals and Assessment table is designed to support preparation of an assessment plan.

	Goal	Ongoing Assessment Opportunities	Unit Assessment Opportunities
Level I: Conceptual and Procedural Knowledge	**1.** Describe and graph directions using compass directions, angles, and direction pairs.	**Section A** p. 3, #3, 4 **Section B** p. 12, #5ab, 6 p. 14, #13abc, 14	**Quiz 1** #1abdce, 3abcd, 4a **Test** #1ab
	2. Understand and graph horizontal and vertical lines and their equations.	**Section A** p. 5, #14abc p. 6, #15a	**Quiz 1** #2c, 3d
	3. Use inequalities to describe regions restricted by horizontal and vertical lines.	**Section A** p. 7, #19	
	4. Find and use equations of the form $y = i + sx$ using the slope and y-intercept.	**Section C** p. 23, #10, 11ab **Section E** p. 41, #11a	**Quiz 2** #1bcd, 2cd **Test** #5ab
	5. Graph points and lines in a coordinate system.	**Section A** p. 5, #11ab **Section E** p. 40, #8a	**Quiz 1** #2abc, 3bc, 4a **Quiz 2** #1ab, 2ab **Test** #2ab, 3abd
	6. Solve equations of the form $a + bx = c + dx$.	**Section D** p. 31, #12abc p. 33, #17ab p. 34, #18bc, **Section E** p. 41, #11b	**Quiz 2** #3 **Test** #4abcd

	Goal	Ongoing Assessment Opportunities	Unit Assessment Opportunities
Level II: Reasoning, Communicating, Thinking, and Making Connections	**7.** Understand the meaning of slope in different contexts.	**Section B** p. 13, #11ab p. 14, #15c p. 16, #18ab, 19ab **Section C** p. 23, #12b	**Quiz 1** #4b **Quiz 2** #1e **Test** #3e
	8. Understand how to find the intersection point of two lines, algebraically and graphically.	**Section B** p. 17, #20b **Section E** p. 40, #4ab, 5, 7	**Test** #2cd, 3c
	9. Understand the graph of a line in the coordinate plane.	**Section B** p. 17, #20ab **Section C** p. 22, #6ab p. 23, #12ac p. 25, #19	**Quiz 2** #1f **Test** #3d

	Goal	Ongoing Assessment Opportunities	Unit Assessment Opportunities
Level III: Modeling, Generalizing, and Non-Routine, Problem Solving	**10.** Model a problem situation and translate it to a graph or an equation.	**Section D** p. 33, #18a p. 37, CYW #5 **Section E** p. 40, #6	
	11. Choose an appropriate way to solve equations.	**Section E** p. 39, #3b p. 42, CYW #3a p. 43, CYW #4	**Test** #5d
	12. Understand the similarities between graphic and algebraic strategies.	**Section E** p. 41, #11c p. 43, For Further Reflection	

Materials Preparation

The following items are the necessary materials and resources to be used by the teacher and students throughout the unit. For further details, see the Section Overviews and the Materials sections of the Hints and Comments column of each teacher page. Note: Some contexts and problems can be enhanced through the use of optional materials. These optional materials are listed in the corresponding Hints and Comments section.

Student Resources

Quantities listed are per student.

- **Letter to the Family**
- **Student Activity Sheets 1–6**

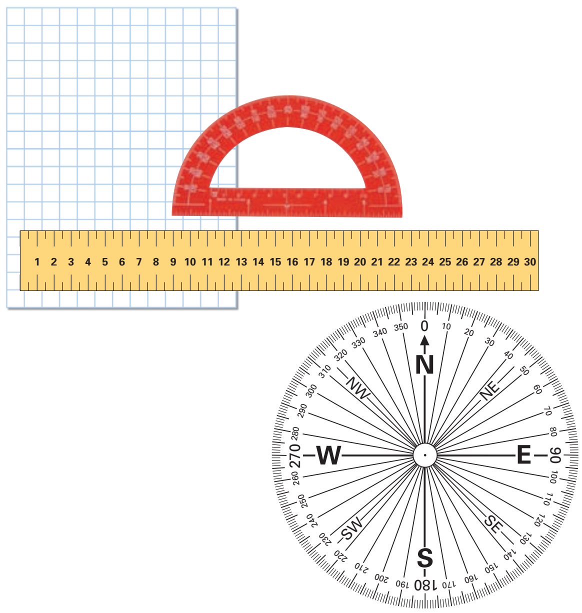
Teacher Resources

No resources required.

Student Materials

Quantities listed are per student, unless otherwise noted.

- Centimeter ruler
- Compass card
- Graph paper (at least five pages, per student)
- Protractor
- Ruler

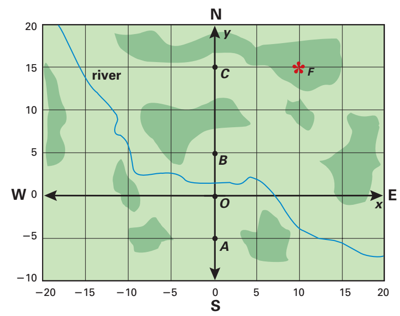


BRITANNICA

Mathematics in Context

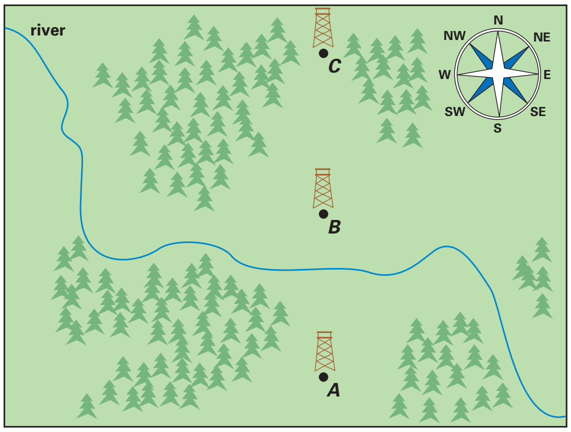
Student Material and Teaching Notes

Contents



Dear Student,

Graphing Equations is about the study of lines and solving equations. At first you will investigate how park rangers at observation towers report forest fires. You will learn many different ways to describe directions, lines, and locations. As you study the unit, look around you for uses of lines and coordinates in your day-to-day activities.



You will use equations and inequalities as a compact way to describe lines and regions.

A "frog" will help you solve equations by jumping on a number line. You will learn that some equations can also be solved by drawing the lines they represent and finding out where they intersect.

We hope you will enjoy this unit.

Sincerely,

The Mathematics in Context Development Team

Section Focus

Students use compass directions and degree measurements to plot lines to locate forest fires on a map. Later in the section, students also use coordinates to locate these fires on a computer screen that uses a four-quadrant coordinate grid. Students explore how the (x, y) positions change as the fire moves in different directions. They use equations to describe the movement of fires along lines. Students draw firebreaks that follow parts of horizontal and vertical lines, and use inequalities to describe regions enclosed by firebreaks.

Pacing and Planning

Day 1: Where's the Fire?		**Student pages 1–4**
INTRODUCTION	Problems 1 and 2	Review compass directions and degree measurements to describe direction.
CLASSWORK	Problems 3–5	Use degree measurements of fire sightings from two fire towers to locate fires and plot coordinate points.
HOMEWORK	Problems 6 and 7	Use given information to identify and plot coordinate points on a map.

Day 2: Coordinates on a Screen		**Student pages 4–5**
INTRODUCTION	Review homework.	Review homework from Day 1.
CLASSWORK	Problems 8–14	Identify and plot coordinate points and introduce the terms: *origin*, *axis*, and *quadrant*. Use equations to represent vertical and horizontal lines.
HOMEWORK	Check Your Work, 1–3	Locate points in a coordinate system.

Day 3: Fire Regions		**Student pages 6–7**
INTRODUCTION	Problems 15 and 16	Use equations to represent vertical and horizontal lines.
CLASSWORK	Problems 17–19	Use inequalities to identify and plot rectangular regions on a grid.
ASSESSMENT	Check Your Work, 4 and 5 For Further Reflection	Student self-assessment: Use equations to describe movement and use inequalities to describe regions.

Additional Resources: *Algebra Tools;* Additional Practice, Section A, pages 44 and 45

Materials

Student Resources
Quantities listed are per student.

- Letter to the Family
- **Student Activity Sheets 1–3**

Teachers Resources
No resources required.

Student Materials
Quantities listed per student.

- Centimeter ruler
- Compass card or protractor
- Ruler

* See Hints and Comments for optional materials.

Learning Lines

Directions and Locations

Describing directions is an important concept in geometry. In this unit, directions are connected to algebra, specifically to straight lines. Directions can be described in words, symbols (N, E, S, W), degrees, and with other measurements such as with coordinates relating to a coordinate grid system, or with slope.

Compass Directions and Degrees

Students are familiar with the eight wind or compass directions such as North (N) or Southwest (SW) from earlier geometry units. This type of direction, starting from a point (the fire tower), is now used to locate forest fires. Students realize that two directions from different starting points (fire towers) are needed to locate a fire. A more refined way of giving directions is to divide the circle even more, into 360°. Students use degrees, beginning with 0° and measuring clockwise up to 360°, to locate fires. Based on given directions, students show the locations of fires on the map by intersecting the lines in the given directions.

Coordinates

The (x, y) notation in a coordinate grid system is another way of finding locations. It differs from the ways for using wind directions and degrees in that it does not give a direction. Students were introduced to the coordinate grid system in the unit *Operations*. Students locate fires and plot and find points with given coordinates in all four quadrants. Students use the formal vocabulary connected to the coordinate system.

Directions and Coordinates

Coordinates and directions are also combined: for a given location in (x, y) notation, students must be able to find the correct directions in degrees from different fire towers. (See, for example, problem 10 on page 4.) Directions involving the coordinate grid instead of degrees will be introduced in the next section, as students are introduced to the concept of slope.

Lines and Regions

Locating fires from fixed positions (the fire towers) provides students with experiences that lead to the equations of vertical and horizontal lines, and later in the unit to slope and equations of other straight lines. The moving of a fire along vertical and horizontal lines leads to equations such as $x = 10$ for a vertical line, which is the set of all points whose x-coordinate is the same, and $y = 8$ for a horizontal one (the y-coordinate is fixed). In other algebra units, students have seen and used equations of lines, as arrow formulas, or written as equations with word-variables. In this unit, the more formal notation using x and y is used. Vertical and horizontal lines forming the boundaries of a rectangular region in which a fire spreads lead to the definition of a region by using inequalities; inequalities can be described in words or by using inequality signs. In this section, a region is described in three ways:

in the context:	*the region north of the firebreak (line) at* $y = 8$*;*
in words:	y *is greater than 8; and*
in symbols:	$y > 8$.

Also two-sided inequalities such as $16 < x < 18$ are used.

At the End of This Section: Learning Outcomes

Students are able to describe and graph directions using wind directions and angles. They understand how to find the intersection point of two lines graphically. Students can also give coordinates of a labeled point in a coordinate grid and reversely show the location of a point with given coordinates. They can describe and graph horizontal and vertical lines using their equations and can use inequalities to describe regions restricted by horizontal and vertical lines.

A Where There's Smoke

Notes

The fire-fighting context is used extensively in this unit. Help students connect to this context through a short class discussion, particularly if students have little or no experience with the woods or forests.

Possible questions: *Have you ever been to a forest? Where? What was it like? Why are forests important? What are some things forest rangers deal with in their job? How might the tower in the picture be used?*

Watch for news items related to forests and forest fires. Forest fires are in the news in many areas, particularly in the late summer and fall.

A Where There's Smoke

Where's the Fire?

From tall fire towers, forest rangers watch for smoke. To fight a fire, firefighters need to know the exact location of the fire and whether it is spreading. Forest rangers watching fires are in constant telephone communication with the firefighters.

Reaching All Learners

Extension

You may want to have students look at a map of a park and try to locate fire towers. They can use this map throughout the unit to relate what they learn to another situation.

English Language Learners

Students sometimes skip the reading to get to the numbered problems. Reading the introductory material as a class can help.

Hints and Comments

Overview

Students are introduced to the context of locating forest fires from fire towers. This context will be used throughout the unit. There are no problems on the page for students to solve.

About the Mathematics

In this section, directions are described using the eight main wind directions (north, northeast, south, southeast, and so on) and degree measurements, where 0 degrees is North and measurements progress clockwise. Some students may remember this notation from the unit *Figuring All the Angles*.

Students also use coordinates to describe locations. They were introduced to the coordinate system in the unit *Operations*. They use and write equations using y and/or x for horizontal and vertical lines and use inequalities to describe rectangular regions.

A Where There's Smoke

Notes

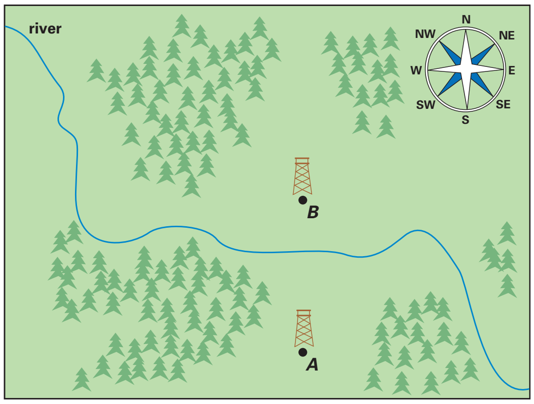
Be sure to have rulers, compass cards, and extra copies of the activity sheets available.

1 If students think the location is specified by one sight line, ask them to draw the line NW from tower *A*. Ask where on the line the fire is located. Students will see that they have narrowed the field but don't know the exact location.

2 Be sure students draw the lines of sight with a ruler and are not just eyeballing the location.

A Where There's Smoke

The map shows two fire towers at points *A* and *B*. The eight-pointed star in the upper right corner of the map, called a **compass rose**, shows eight directions: north, northeast, east, southeast, south, southwest, west, and northwest. The two towers are 10 kilometers (km) apart, and as the compass rose indicates, they lie on a north-south line.



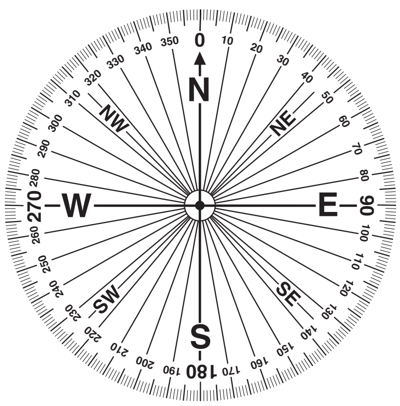
One day the rangers at both fire towers observe smoke in the forest.

The rangers at tower *A* report that the smoke is directly northwest of their tower.

1. Is this information enough to tell the firefighters the exact location of the fire? Explain why or why not.

The rangers at tower *B* report that the smoke is directly southwest of their tower.

2. Use **Student Activity Sheet 1** to indicate the location of the fire.



In problems 1 and 2, you used the eight points of a compass rose to describe directions. You can also use **degree measurements** to describe directions.

A complete circle contains 360°. North is typically aligned with 0° (or 360°). Continuing in a clockwise direction, notice that east corresponds with 90°, south with 180°, and west with 270°.

You measure directions in degrees, clockwise, starting at north.

Reaching All Learners

Intervention

Before students begin problem 1, make sure that they understand how to read the map. Students should do these problems by drawing straight lines using a ruler and the directions indicated in the compass rose on the map.

Vocabulary Building

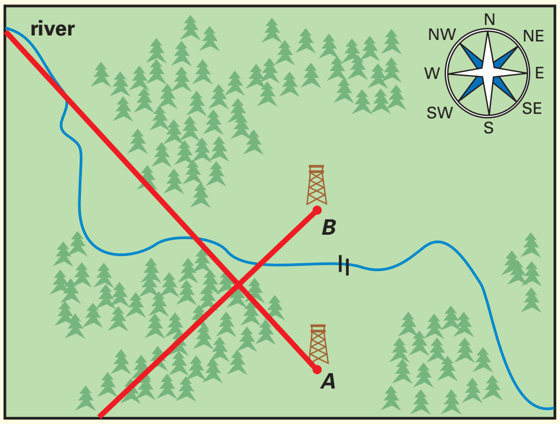
You may wish to create a vocabulary section in student notebooks. Have them add the terms *compass rose* and *degree measurements* to the the notebook. It is sometimes helpful to have students give examples or draw illustrations in this section.

English Language Learners

Discuss with English language learners the context of forest rangers watching for fires from towers.

Solutions and Samples

1. No. There are many points on the straight line that leads northwest from tower *A*.
2. Students should draw lines from tower *A* to the northwest and from tower *B* to the southwest. The intersection point of the two lines is the location of the fire, as the map below shows.



Hints and Comments

Materials

Student Activity Sheet 1 (one per student);
rulers (one per student);
compass cards (one per student)

Overview

Students use a compass rose and a ruler to locate a fire on a map.

About the Mathematics

The eight main directions shown on the compass rose can be further refined, for example, into sixteen directions, maybe including NNE and WSW. In this unit, however, only the eight major wind directions are used.

On this page, the use of degrees instead of compass directions to indicate directions is introduced. The four main compass directions are connected to 0° (or 360°) north; 90° east; 180° south; and 270° west.

Planning

Students may work on problems 1 and 2 individually.

Comments About the Solutions

1. Students should realize that one direction from one point is not enough information to locate a second point; the second point could be anywhere on the ray that moves in that direction.
2. When a direction from another point is introduced, the location of the fire can be determined.

Did You Know?

From a tower 10 m high, you can see about 36 km. In general, you can use the following rule: If a tower is h m tall, the distance you can see is 3.6 h km.

Notes

3 Have students draw a north-south line through the towers before they begin to orient the compass card accurately.

3 and **4** Students must measure clockwise starting at 0° north. Small differences between students' answers are acceptable, but students should try to measure as accurately as possible.

5 Before students begin working on problem 5, you may want to discuss the main features of the map. Have students locate point *O* and towers *A*, *B*, and *C*. Be sure to discuss the scale of the map. Notice that the map on the previous page shows a smaller part of the park than what is displayed here.

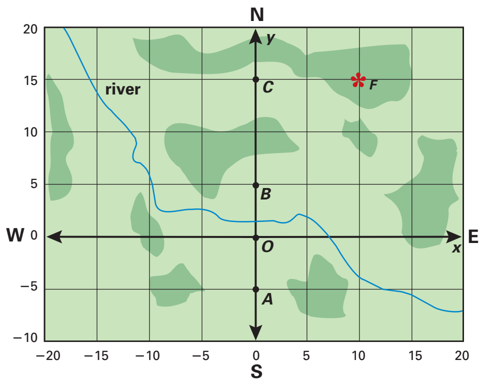
Smoke is reported at 8° from tower *A*, and the same smoke is reported at 26° from tower *B*.

3. Use **Student Activity Sheet 2** to show the exact location of the fire.

4. Use **Student Activity Sheet 2** to show the exact location of a fire if rangers report smoke at 342° from tower *A* and 315° from tower *B*.

Coordinates on a Screen

The park supervisor uses a computerized map of the National Park to record and monitor activities in the park. He also uses it to locate fires.



The computer screen on the left shows a map of the National Park. The shaded areas indicate woods. The plain areas indicate meadows and fields without trees. The numbers represent distances in kilometers.

Point *O* on the screen represents the location of the park supervisor's office, and points *A*, *B*, and *C* are the rangers' towers.

5. **a.** What is the distance between towers *A* and *B*? Between tower *C* and point *O*?

   b. How is point *O* related to the positions of towers *A* and *B*?

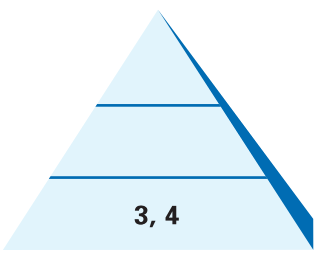
A fire is spotted 10 km east of point *C*. The location of that point (labeled *F*) is given by the coordinates 10 and 15. The coordinates of a point can be called the **horizontal coordinate** and the **vertical coordinate**, or they can be called the ***x*-coordinate** and the ***y*-coordinate**, depending on the variables used in the situation.

$F = (10, 15)$

horizontal coordinate or *x*- coordinate

vertical coordinate or *y*- coordinate

Assessment Pyramid



Use direction lines to find a location.

Reaching All Learners

Intervention

You may want to model the steps for drawing the line of sight for 3 on an overhead, with students completing the steps with you on their activity sheets. Ask students to explain the reason for each step.

English Language Learners

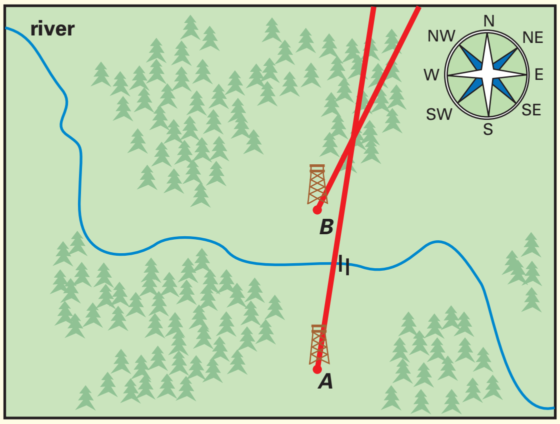
Ask students to summarize the steps for finding the location of the fire either verbally to a partner or in writing.

Vocabulary Building

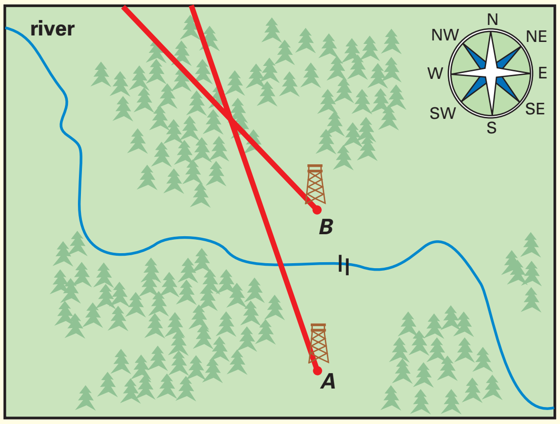
Have the students add the terms *horizontal coordinate*, *vertical coordinate*, *x-coordinate*, and *y-coordinate* to the vocabulary section of their notebooks.

Solutions and Samples

3.



4.



5. a. The distance between towers *A* and *B* is 10 km. The distance between tower *C* and point *O* is 15 km.

b. Point *O* is halfway between towers *A* and *B*.

Hints and Comments

Materials

Student Activity Sheet 2 (one per student); compass cards (one per student); protractors, (optional, one per student)

Overview

Students are introduced to the context of this section, computerized mapping of National Park activities. The computer map that is used in the rest of this section is presented on this page. Students also use a compass card, which they may remember from the unit *Figuring All the Angles* to locate more fires in the park.

About the Mathematics

The advantage of a map with a grid over a map without a grid is that places on a grid map are easier to locate. A grid (or coordinate system) usually consists of two axes (a horizontal axis and a vertical axis) and scale marks that are at fixed intervals along the axes. In this way distances can be used as well. The intersection of the two axes is called the *origin*.

Planning

You may want to review how to use a compass card in a brief class discussion. Students may work on problems 3 and 4 in small groups.

Extension

You may want to ask students to choose a location for the fire, and then find the angle at which the fire is seen from each tower.

The following problem can also be used as an extension problem with **Student Activity Sheet 3**.

There is a new tower at point *C*. It is 10 km “due north” of tower *B*, as shown below. The fire fighters receive reports of smoke that is 294° from tower *A*, 247° from tower *B*, and 210° from tower *C*.

a. The fire fighters know that something is wrong with these reports. Explain how they know.

b. Further reports confirm that the observations from towers *A* and *C* are correct, but the observation from tower *B* is incorrect. Find the correct observation to report from tower *B*.

Notes

6 and **7** Check that students write the *x*-coordinate first in the pair. Reinforce that parentheses are used to indicate a coordinate pair.

9 Some students may give a general response, such as, "The point is left of *A*." Ask, *What direction from A? How far?*

10 Have students put a ruler of the edge of paper on the line from the tower to the fire so it is easy to read the direction on the compass card. Check that students center the compass card on the towers, not on the fire.

Use the map on page 3 to answer problems 6 and 7.

6. a. Find the point that is halfway between *C* and *F*. What are the coordinates of that point?

b. Write the coordinates of the point that is 10 km west of *B*.

The coordinates of fire tower *B* are (0, 5).

7. a. What are the coordinates of the fire towers at *C* and at *A*?

b. What are the coordinates of the office at *O*?

The rangers' map is an example of a **coordinate system**. Point *O* is called the **origin** of the coordinate system. If the coordinates are written as (x, y):

the horizontal line through *O* is called the ***x*-axis**.

the vertical line through *O* is called the ***y*-axis**.

The two axes divide the screen into four parts: a northeast (NE) section, a northwest (NW) section, a southwest (SW) section, and a southeast (SE) section. Point *O* is a corner of each section, and the sections are called **quadrants**.

8. The coordinates of a point are both negative. In which quadrant does the point lie?

Use the map on page 3 to answer problems 9 and 10.

9. Find the point $(-20, -5)$ on the computer screen on page 3. What can you say about the position of this point in relation to point *A*?

There is a fire at point *F* (10, 15).

10. What directions, measured in degrees, should be given to the firefighters at towers *A*, *B*, and *C*?

Reaching All Learners

Vocabulary Building

Quite a bit of vocabulary is introduced on this page. Have students keep a list of definitions with illustrations. Include *coordinate system*, *x-axis*, *y-axis*, *horizontal*, *vertical*, *origin*, and *quadrant*.

Act It Out

To help students remember *x*- and *y*-coordinates, try these hand motions and chant. Cross your arms in front of your chest and open them to form a horizontal line, while saying, *x goes across*. Raise your arms to form a Y with the body, while saying, *y goes high*.

Solutions and Samples

6. a. (5, 15)

b. (–10, 5)

7. a. tower C: (0, 15)

tower A: (0, –5)

b. office at O: (0, 0)

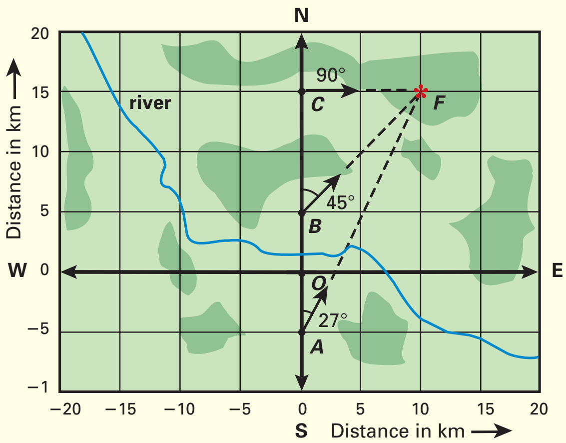
8. The southwest quadrant.

9. The point is 20 km west of tower A.

10. tower A: 27°

tower B: 45°

tower C: 90°



Hints and Comments

Materials

compass cards (one per student);
local area or park maps, (optional, one per student)

Overview

Students are introduced to the formal terms *coordinate system, origin, quadrants, x-axis,* and *y-axis.* Students determine the locations of fires relative to each tower.

About the Mathematics

Directions and coordinates are related to each other. On this page, this relationship is addressed informally. Later in the unit, students will learn how to relate angles to coordinates, using slope and the tangent ratio. Coordinates and the coordinate grid system have been introduced and used in earlier *Mathematics in Context* units. The coordinate system with four quadrants was formally introduced in *Operations.*

Comments About the Solutions

6. and **7.**
Students should use the map on page 3 of the Student Book to answer the questions.

6. a. Since point F and tower C have the same y-coordinate, the point halfway between them also has a y-coordinate of 15.

6. b. Students should know that horizontal directions correspond to east–west and that vertical directions correspond to north–south.

8. and **9.**
Before students begin working on these problems, you may want to review the coordinate system terminology on page 4.

8. This quadrant is also called *the third quadrant* in most mathematics texts.

Extension

You may want to have students draw a coordinate grid system on a map of their own and find the coordinates of some points on the map. Encourage students to use natural points as the centers of their maps. These central points should have the coordinates (0, 0).

Where There's Smoke

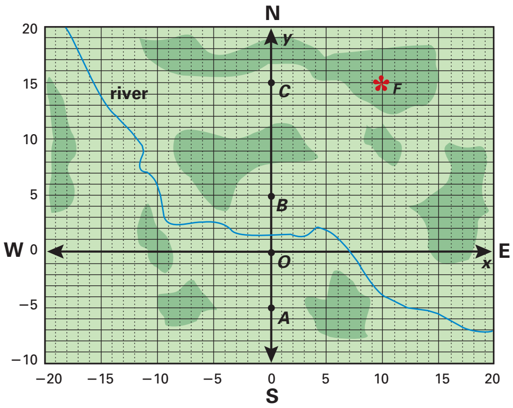
Notes

11b You may need to remind students that coordinates do not need to be whole numbers! Accept reasonable estimates here.

12a It is helpful to some students to use a finger to trace the path of the fire as it moves and then identify the points.

12a Some students think "after 10 more km south" is from the original location and some add 10 km to the 3 km already moved. This difference is not too important, as long as the students correctly identify the point they are using.

13 After students work on this question, discuss this as a whole class. Students may have different ways of seeing this.



The computer screen can be refined with horizontal and vertical lines that represent a grid of distances 1 km apart. The side of each small square represents 1 km.

The screen on the left shows a river going from NW to SE.

11. **a.** What are the coordinates of the two points where the river leaves the screen?

b. What are the coordinates of the points where the river crosses the x-axis? Where does it cross the y-axis?

A fire is moving from north to south along a vertical line on the screen. The fire started at F (10, 15).

12. **a.** What are its positions (coordinates) after the fire has moved 1 km south? After it has moved 2 km south? After 3 km south? After 10 more kilometers south?

b. Describe what happens to the x-coordinate of the moving fire.

Vertical and horizontal lines have special descriptions. For example, a vertical line that is 10 km east of the origin can be described by $x = 10$.

13. **a.** Why does $x = 10$ describe a vertical line 10 km east of the origin?

b. How would you describe a horizontal line that is 5 km north of point O? Explain your answer.

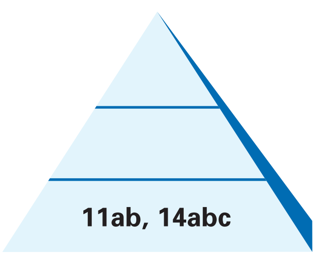
14. **a.** Where on the screen is the line described by $x = -5$?

b. Where on the screen is the line described by $y = 15$?

c. Describe the path of a fire that is moving on the line $y = 8$.

The description $x = 10$ is called an **equation of the vertical line** that is 10 km east of O. An **equation of the horizontal line** that is 10 km north of O is $y = 10$.

Assessment Pyramid



Graph points.

Locate horizontal and vertical lines given equations.

Reaching All Learners

Accommodation

Some students may benefit from having an extra copy of **Student Activity Sheet 3** to use with problems 11–14.

Act It Out

Place number strips on the floor or in the schoolyard at right angles to form axes. Give students cards with coordinates; have students find and stand on the indicated locations. Students can exchange cards and repeat. Alternatively, students can work in pairs. One finds a location and the other names the location with its coordinates. Students change roles and repeat.

Vocabulary Building

Have students add *equation* and *equation of vertical and horizontal lines* to their notebooks, with examples for each.

Solutions and Samples

11. a. The river leaves the screen at coordinates $(-18, 20)$ and $(20, -7)$.

b. The river crosses the x-axis at $(7, 0)$ and the y-axis at approximately $(0, 1.5)$.

12. a. (10, 14) after 1 km

(10, 13) after 2 km

(10, 12) after 3 km

(10, 2) after 10 more km

b. The x-coordinate stays the same.

13. a. Answers will vary. Some students may say that $x = 10$ is a vertical line that crosses the x-axis at 10. Another answer would be that every point on the line has an x-coordinate of 10, which means that the x-coordinate is fixed.

b. $y = 5$. Explanations will vary.

Sample explanation: All points on this horizontal line have a y-coordinate of 5.

14. a. The line $x = -5$ is a vertical line crossing the x-axis 5 km west of the y-axis, (that is at point $(-5, 0)$).

b. The line $y = 15$ is a horizontal line crossing the y-axis 15 km north of the x-axis, (that is at the point (0,15)).

c. Answers may vary. Sample answer: The fire moves in a horizontal direction 8 km north of the x-axis.

Hints and Comments

Materials

Student Activity Sheet 3 (optional)

Overview

Students determine the locations of fires relative to each tower and the locations where a river leaves the screen or intersects the axes. They investigate what happens to the coordinates of a fire when it moves horizontally and vertically.

About the Mathematics

The place where the river crosses one of the axes of the graph is called the x-intercept (crossing the x-axis) or the y-intercept (crossing the y-axis). The term y-intercept will be introduced formally in Section C. When something moves vertically, the x-coordinate stays the same. When something moves horizontally, the y-coordinate stays the same. The equations for horizontal and vertical lines are introduced. Later in this unit, the equations for other lines will be introduced.

Comments About the Solutions

11. b. The answer given in the Solutions column for the point at which the river crosses the y-axis is approximate. Accept answers that are reasonably close to the given answer.

12. a. Make sure students understand that the fire is moving continuously along the entire line of points with x-coordinates that are equal to 10.

13. Problem 13 is critical because this is where the equations of horizontal and vertical lines are introduced. For part **a**, students may answer, "because it crosses the x-axis at $x = 10$," which shows some understanding. There is no need to teach a more formal understanding of this concept at this time.

14. The equations are connected to the descriptions of the moving fire. Students can describe the location of lines on the screen by using words that have a meaning in the context.

A Where There's Smoke

Notes

15a Observe students as they work to check that they have drawn the correct lines.

15a If students have trouble drawing the horizontal and vertical lines from the equations, ask some leading questions. For example, ask student to point to a place where $x = 14$. Students usually point to (14, 0). Ask for another point where $x = 14$. If the students are stuck, ask for a point where the x-coordinate is 14 but the y-coordinate is not 0.

16 Share student explanations for **16a** and **b** in whole class discussion.

Fire Regions

To prevent forest fires from spreading, parks and forests usually contain a network of wide strips of land that have only low grasses or clover, called *firebreaks*. These firebreaks are maintained by mowing or grazing.

In the forest, some firebreaks follow parts of the lines described by the equations $x = 14$, $x = 16$, $x = 18$, $y = 8$, $y = 6$, $y = 4$, $y = 2$, and $y = 0$.

15. **a.** Using **Student Activity Sheet 3**, draw the firebreaks through the wooded regions of the park.

b. Write down the coordinates of 5 points that lie north of the firebreak described by $y = 8$.

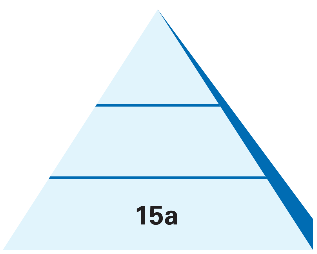
The fire rangers describe the region north of the firebreak at $y = 8$ with "y is greater than 8." This can be written as the *inequality* $y > 8$.

16. **a.** Explain how $y > 8$ describes the whole region north of $y = 8$.

b. Why is it not necessary to write an inequality for x to describe the region north of $y = 8$?

c. Describe the region west of the firebreak at $x = 14$ by using an inequality.

Assessment Pyramid



Graph horizontal and vertical lines given equations.

Reaching All Learners

Intervention

For additional practice with coordinates:

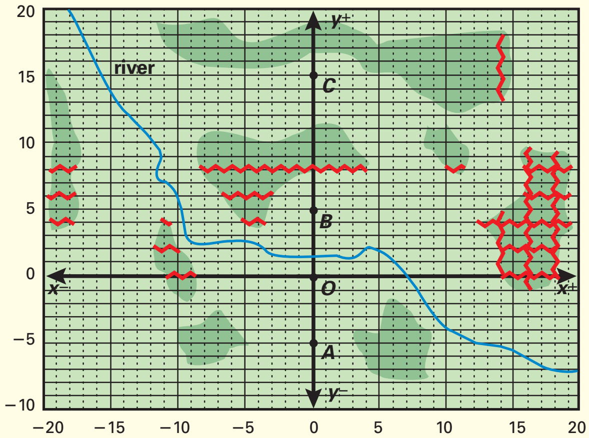
- Play a 20-questions game where students try to find a secret point on a grid.
- Give coordinates of points that students locate on a grid and connect to make a simple picture.

Extension

Ask students to create pictures using straight lines on a four-quadrant grid. The students then identify the vertices using coordinates. The list of coordinates can be given to a classmate or parent to create the picture.

Solutions and Samples

15. a.



Students may draw the lines that describe the firebreaks as well; these run also through the non-wooded areas

b. Answers will vary, sample answers (0, 9) (0, 10) (1, 10) (−5, 15) (10, 15). Coordinates are correct if the y-coordinate is greater than 8.

16. a. Answers may vary. Sample answer: $y > 8$ means that the vertical coordinate is greater than 8; all these points lie above the line $y = 8$, on which lie all points with vertical coordinates equal to 8.

b. Answers will vary. Sample answer: The x-coordinate does not matter for this region. All values for x are okay.

c. $x < 14$.

Hints and Comments

Materials

Student Activity Sheet 3 (one per student);
centimeter rulers (one per student);
transparency of **Student Activity Sheet 3,** (optional, one per class);
overhead projector, (optional, one per class)

Overview

Students use equations to describe horizontal and vertical lines, and review the use of inequalities to describe a region.

About the Mathematics

Students were introduced to inequality signs in the unit *Operations*. Here students are introduced to the use of inequalities to describe rectangular regions that have horizontal and vertical lines as boundaries.

Comments About the Solutions

15. a. If students have difficulties, you might make a transparency of the map on **Student Activity Sheet 3** in order to point out the regions through which the firebreaks will go. Students may first want to draw the whole lines as they fit on the screen. This means that they also run through the non-wooded area. They can mark the parts through the wooded regions, which form the actual firebreaks later. You may want to discuss why firebreaks only exist in the wooded areas.

15. b. Have students discuss what makes the coordinates of a point be a correct solution for this problem: They only need to look at the y-coordinate! You may want to have students check if their own points are visible on the screen.

16. Problem 16 is critical since it introduces the meaning of an inequality as being a region bounded by a line. If students have trouble writing inequalities, have them describe the x-coordinates in the region. Ask for the smallest x-coordinate in the region and the largest. How can this range be described using an inequality? If necessary, ask what two numbers the x values are between. Monitor the students, using the same procedure for the y-coordinate. When done, ask students to summarize the procedure in their own words, either orally or in writing.

A Where There's Smoke

Notes

Careful reading of this text is critical! This is new information. Read this text aloud in small groups or as a whole class to help weaker readers.

A clean copy of **Student Activity Sheet 3** should be used for problems 17 though 19.

17 For students who have trouble, have them put their pencils on the point (17, 5). Remind them that the fire can travel in any direction until it reaches a firebreak. Ask, *Where can it go?*

19 It may help students to do this in two steps. First, locate the strip where $-6 < x < -3$ and then the strip where $6 < y < 10$. The requested region is where these strips overlap.

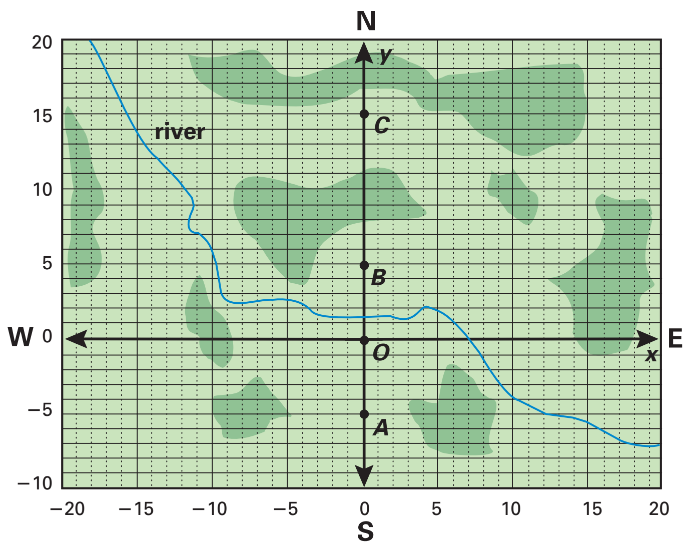
A fire is restricted by the four firebreaks that surround it. If a fire starts at the point (17, 5), then the vertical firebreaks at $x = 16$ and $x = 18$ and the horizontal firebreaks at $y = 4$ and $y = 6$ will keep the fire from spreading. Here is one way to describe the region:

x is between 16 and 18; *y* is between 4 and 6.

You can use inequalities to describe the region:

$16 < x < 18$ and $4 < y < 6$

This can also be read "*x* is greater than 16 and less than 18, and *y* is greater than 4 and less than 6."



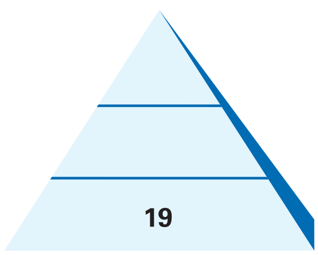
Use **Student Activity Sheet 3** for problems 17 through 19.

17. Show the restricted region for a fire that starts at the point (17, 5).

18. Another fire starts at the point (15, 3). The fire is restricted to a region by four firebreaks. Show the region and use inequalities to describe it.

19. Use a pencil of a different color to show the region described by the inequalities $-6 < x < -3$ and $6 < y < 10$.

Assessment Pyramid



Use inequalities to describe regions.

Reaching All Learners

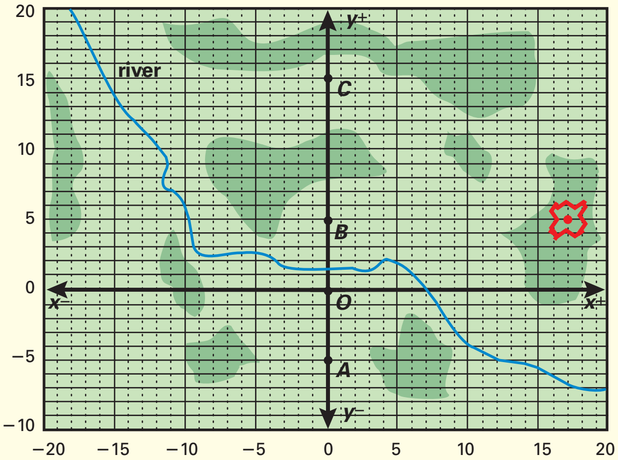
Intervention

Students often have trouble deciding if a horizontal or vertical line should be $x = c$ or $y = c$. Possible reasons for this confusion: (1) A line parallel to the *y*-axis has the equation $x = c$. (2) the *x*-coordinate tells how far over a point is in the horizontal direction, but a horizontal line has equation $y = c$. Ask students to explain why this is true.

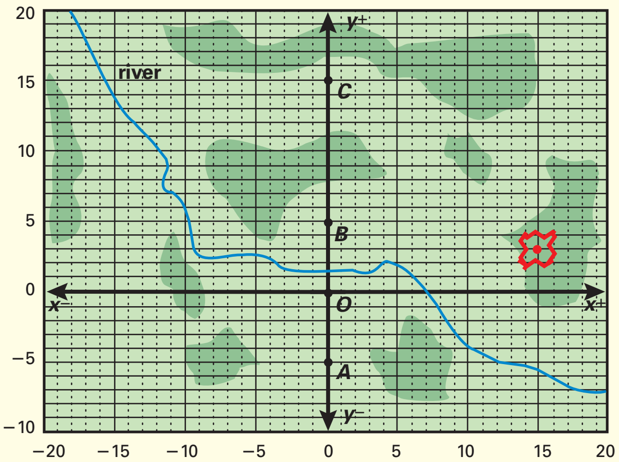
Focus students on the location of the line relative to the origin. A vertical line is located left or right of the origin; its equation is $x = c$. Another idea is to name several points on the line. Ask, *What do they have in common?*

Solutions and Samples

17.

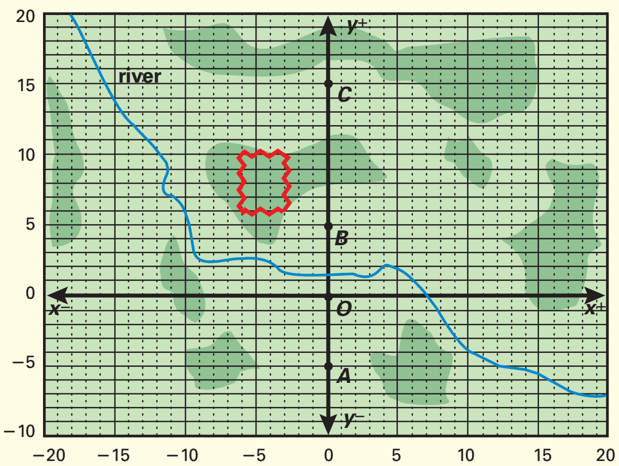


18. The region is $14 < x < 16$ and $2 < y < 4$ or in words: x is between 14 and 16, and y is between 2 and 4.



As a help, students may have drawn the whole lines describing the firebreaks (through the non-wooded areas) as well.

19.



As a help, students may have drawn the whole lines describing the firebreaks (through the non-wooded areas) as well.

Hints and Comments

Materials

Student Activity Sheet 3 (one per student);
centimeter rulers (one per student);
transparency of **Student Activity Sheet 3**, (optional, one per class);
overhead projector, (optional, one per class)

Overview

Students review the use of inequalities to describe a region.

Comments About the Solutions

17.–19. You may want to make a transparency of **Student Activity Sheet 3** so that students can show the restricted regions of the fires on the overhead projector. You may also want to remind students that there are firebreaks only in wooded areas.

18. Students may use either the equations or the words to describe the region. They may want to draw the whole lines—also through non-wooded areas—that describe the firebreaks as well.

It might be worth mentioning here the distinction between using the < and > signs and the ≤ and ≥ signs to describe regions.

If students wanted to describe the region inside the boundaries, they should use the "greater than" (>) or "less than" (<) symbols. If students wanted to include the fire breaks when describing the regions, they would need to use the "greater than or equal to" (≥) or "less than or equal to" (≤) signs.

19. If students have difficulty showing the region on the map, they may want to write the region described by the equations first in words as is done at the beginning of Student Book page 7. They can then draw the whole lines that describe the firebreaks first and then find the region.

Extension

You may want to have students make and describe regions on their own maps. Students can use the same maps that they used for the previous Extension activity in this section.

Where There's Smoke

Notes

Students often think they can skip the Summary, since there are no problems attached.

Some suggested strategies to overcome this:

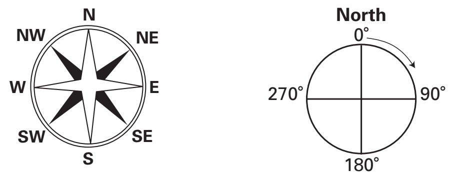
- Read the Summary aloud as a class. Ask volunteers to read a paragraph.
- Ask students to write a sentence to summarize each paragraph. This can be an exit ticket from class.
- Ask students how the Summary might be helpful.

Where There's Smoke

Summary

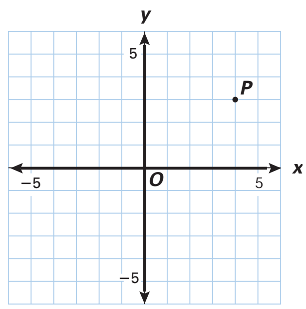
You have seen two ways to indicate a direction starting from a point on a map.

- Using a compass rose, you can indicate one of the eight directions: N, NE, E, SE, S, SW, W, and NW.
- You can indicate direction using degree measurements, beginning with 0° for north and measuring clockwise up to 360°.



Another way to describe locations on a map is by using a grid or coordinate system. In a coordinate system, the horizontal axis is called the *x-axis* and the vertical axis is called the *y-axis*. The axes intersect at the point (0, 0), called the *origin*.

The location of a point is given by the *x*- and *y*-coordinates and written as (x, y).



When points are on a vertical line, the *x*-coordinate does not change. Vertical lines can be described by equations such as $x = 1$, $x = 8$, and $x = -3$.

When points are on a horizontal line, the *y*-coordinate does not change. Horizontal lines can be described by equations such as $y = -5$, $y = 0$, and $y = 3$.

Inequalities can be used to describe a region. For example, $1 < x < 3$ and $-2 < y < 3$ describes a 2-by-5 rectangular region.

Reaching All Learners

Study Skills

Before reading the Summary, ask students to identify three ideas from this section that were new to them. This helps students think about what they have learned and also gives you some valuable insights.

Vocabulary Building

Assign student groups to create a mind map or concept web highlighting the ideas and vocabulary of the unit. Students can present these to their classmates. Post work in the classroom or hallway.

Hints and Comments

Materials

graph paper for optional writing opportunity,
(one sheet per student)

Overview

Students read the Summary, which reviews the main concepts covered in this section. These are: the two ways to indicate a direction starting from a point on a map; the use of a coordinate system to describe locations on a map; the use of equations and inequalities to describe horizontal and vertical lines and rectangular regions on a map.

Writing Opportunity

You may want to have students draw a coordinate system on a sheet of graph paper. Then have them outline some regions and describe them using inequalities. Ask students to write a paragraph or two in their notebooks to explain all the features of the graph.

A Where There's Smoke

Notes

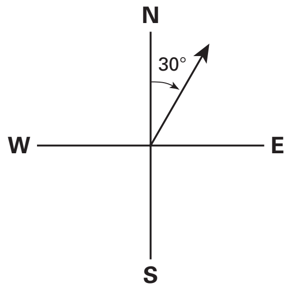
1 Some students will use the compass card and look at the opposite side. This reinforces the idea that the opposite direction is on the same line. Some will add or subtract 180° from the given measure.

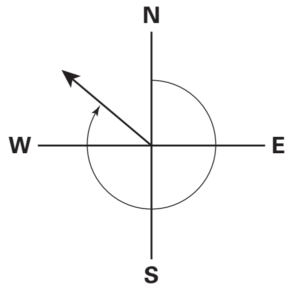
2 You may want to remind students that when a wind blows *from* the northeast, it blows *to* the opposite direction, southwest. Students will need to refer to the map on **Student Activity Sheet 3.**

2b Students should draw the lines from the towers to the fire on the activity sheet or lay the edge of their ruler on the map in the book in order to measure the direction accurately.

3 This question provides the opportunity to reinforce the concept of parallel lines.

Check Your Work





1. a. The direction 30° is shown in the diagram above on the left. What direction is opposite 30°?

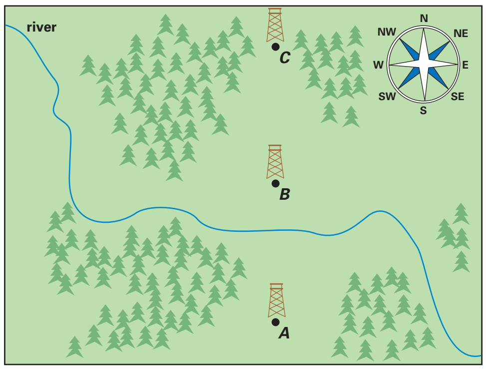
b. What direction is shown above on the right? What degree measurement is the opposite of that direction?

A fire starts at point *F* (10, 15). A strong wind from the NE blows the fire to point *G*, which is 5 km west and 5 km south of *F*.

Note: You can the use the map on page 5 to see the situation.

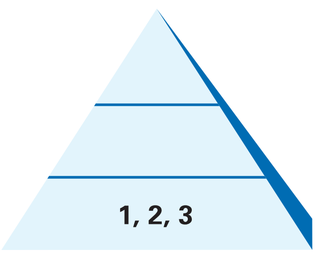
2. a. What are the coordinates of *G*?

b. What directions in degrees will fire towers *A* at (0, −5) and *C* at (0, 15) send to the firefighters?



3. One day, rangers report smoke at a direction of 240° from tower *A* and 240° from tower *B*. Is it possible that both reports for the same fire are correct? Why or why not?

Assessment Pyramid



Assesses Section A Goals

Reaching All Learners

Accommodation

Provide students a copy of **Student Activity Sheet 3** to use with questions 2 and 3. You may want to include a copy of the grid for questions 4 and 5.

Solutions and Samples

Answers to Check Your Work

1. **a.** 210°

   b. 310°, 130°

2. **a.** (5, 10)

   b. Tower *A*: 18°
   Tower *C*: 135°

3. No, at least one report must be incorrect, because the lines going in the same direction are parallel. Parallel lines do not intersect.

Hints and Comments

Materials

Student Activity Sheet 3 (optional)

Overview

Students use the Check Your Work problems for self-assessment. A student who can answer the questions correctly has understood enough of the concepts taught in the section to be able to start the next section. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work. Answers are provided in the Student Book. Have students discuss their answers with classmates.

About the Mathematics

At the end of this section, students should be able to solve problems involving directions on a map as well as problems in which they use the coordinates of points or equations of horizontal and vertical lines and inequalities for regions.

Comments About the Solutions

3. You may want to discuss the possibility that the fire is far away.

A Where There's Smoke

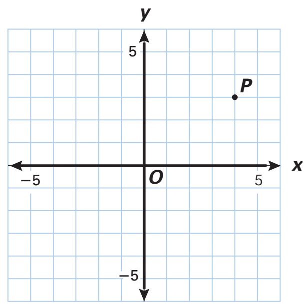
Notes

5 This question involves several steps. If students have trouble, it might help to have them locate the corners of the rectangle and label these points with their coordinates. It may help some students to draw the rectangle on graph paper.

For Further Reflection

It may help students to start with examples and develop the explanation from them.

A Where There's Smoke



4. a. Suppose point P in the coordinate system on the left moves on a straight line in a horizontal direction. What is an equation for that line?

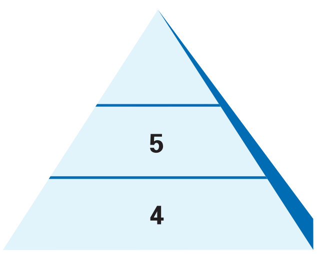
b. Use an inequality to describe the region below the line.

5. In the coordinate system above, point O is the center of a rectangular region, and P is one corner. The boundaries of the region are horizontal and vertical lines. Use inequalities to describe the region.

For Further Reflection

Compare the two ways to indicate a direction starting from a point on a map. Give one advantage of each.

Assessment Pyramid



Assesses Section A Goals

Reaching All Learners

Intervention

Some students may still need assistance writing equations for horizontal and vertical lines. Ask them to review the steps they used before or assist by asking leading questions. Some students find it helpful to move their finger on the line and identify points. Some students may need similar support writing inequalities.

Parent Involvement

You may wish to have students show parents their work from this section and explain how they did the Check Your Work questions.

Solutions and Samples

4. a. $y = 3$

b. The inequality is $y < 3$.

5. The following lines are the boundaries of the region.

$y = -3$ and $y = 3$, so $-3 < y < 3$
$x = -4$ and $x = 4$, so $-4 < x < 4$

For Further Reflection

Sample response:

To indicate directions from a starting point, you could use compass directions, like east or southwest, or use degrees. Compass directions are good to use if you don't have a measuring tool. Degree measurements are more precise.

Hints and Comments

Overview

Students use the Check Your Work problems for self-assessment. The answers to these problems are also provided in the Student Book.

Check Your Work Problems

These problems are designed for student self-assessment. A student who can answer the questions correctly has understood enough of the concepts taught in the section to be able to start the next section. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work. Answers are provided in the Student Book. Have students discuss their answers with classmates.

Section Focus

In the previous section, students located fires using directions on a compass rose or using degrees. They also used (x, y) notation on a coordinate grid map to indicate the location of a fire. The fires were all located relative to the origin (0, 0). In this section, students describe directions in a grid map by using a horizontal and a vertical component. Students start by giving directions using pairs of numbers. They then learn to describe the direction as the slope of a line using a single number, the ratio of the vertical component to the horizontal component.

Pacing and Planning

Day 4: Directing Firefighters		**Student pages 11–13**
INTRODUCTION	Problems 1–6	Use direction pairs to describe direction and discover that more than one direction pair can describe the same direction.
CLASSWORK	Problems 7–9	Explore the use of direction pairs on a coordinate grid.
HOMEWORK	Problems 10 and 11	Investigate direction pairs that describe the same direction and different directions.

Day 5: Comparisons		**Student page 14**
INTRODUCTION	Review homework.	Review homework from Day 4.
CLASSWORK	Problems 12–14	Investigate direction pairs that are opposite and discover that they form one line.

Day 6: Up and Down the Slope		**Student pages 15–17**
INTRODUCTION	Problems 15 and 16	Introduce and apply the concept of slope.
CLASSWORK	Problems 17–19	Determine the slope of a graphed line and graph a line given the slope and a starting point.
HOMEWORK	Problem 20	Determine the slope of graphed lines and use the slope to determine the point at which the lines meet.

Day 7: Summary		Student pages 17–20
REVIEW	Review homework.	Review homework from Day 6.
CLASSWORK	Check Your Work For Further Reflection	Student self-assessment: Use direction pairs and draw lines in the coordinate plane.
ASSESSMENT	Quiz 1	Assesses Section A and B Goals.

Additional Resources: *Algebra Tools*; Additional Practice, Section B, page 46

Materials

Student Resources
Quantities listed are per student.

- **Student Activity Sheets 4 and 5**

Teachers Resources
No resources required.

Student Materials
Quantities listed are per student.

- Graph paper
- Ruler

* See Hints and Comments for optional materials.

Learning Lines

Directions as Pairs of Numbers

In the previous section of this unit, directions were given using wind directions or angle measures in degrees. In the direction pair [+10, +5], the first number gives the horizontal component of the direction, and the second number gives the vertical component. In the context of the forest fire, this direction pair means "go 10 km east and 5 km north." Directions are not given relative to the origin (0, 0) but are relative to any starting point.

Different direction pairs describe the same direction, for example, [+10, +5] and [+20, +10]. Both positive and negative numbers as well as rational numbers are used in direction pairs. In order to avoid confusion between coordinate pairs that indicate a location relative to the origin (0, 0), the notation for direction pairs is different from the notation for coordinate pairs. Direction pairs use "straight" brackets [like these] and a sign for each number.

Since all direction pairs that indicate the same (or opposite) direction have the same ratio, this ratio can be used to describe the direction or slope of a line. Slope is introduced as the ratio of the vertical component of a direction pair divided by the horizontal component.

Directions, Slope, and Equivalent Fractions

The fact that different direction pairs indicate the same direction and thus also must generate the same ratio or slope can be used to address the notion of equivalent fractions. For example, using direction pairs, students should be able to show why $\frac{4}{6} = \frac{-4}{-6}$ or why $\frac{-4}{2} = -2$.

Lines and Slope

The slope of a line gives the "direction" of a line, which is a measure of how steep it is. It is given as the ratio of two numbers, the vertical change over the horizontal change. A line's slope is the same regardless of what two points on the line are used to compute it or what direction pair is used to find it. Students have to find the slope of a line from two given points or from a drawing. Students can try to find a direction pair for a line first and use this to calculate the slope. Students must draw a line if the slope and a point on the line are given. In Section C, the slope component for the equation of a line is discussed.

At the End of This Section: Learning Outcomes

Students will understand that different direction pairs can indicate the same direction and will produce the same slope. Students will understand the meaning of *slope* in different contexts. They will be able to find the slope using a direction pair, two points on a line, and/or a drawing. Students will also be able to draw a line if a direction pair or the slope and a point on the line are given.

B Directions as Pairs of Numbers

Notes

Be sure students read this section carefully. A new concept and new notation are introduced.

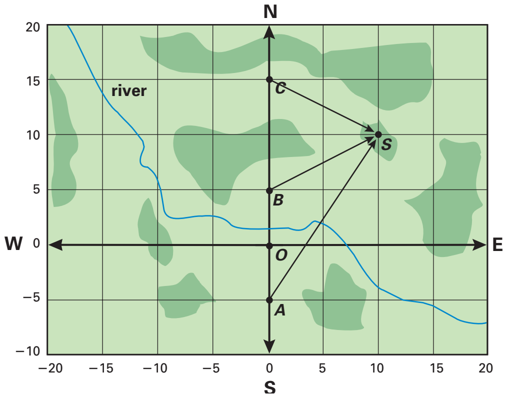
You may want to introduce this section by reviewing how to indicate directions by using wind directions and degree measurements.

B Directions as Pairs of Numbers

Directing Firefighters

In the previous section, directions from a point were indicated by compass references, such as N or NW. A second way to indicate directions involved using degrees measured clockwise from north, such as 30° or 210°. This section introduces a third method to indicate directions.

Smoke is reported at point *S* (10, 10). A firefighting crew is at tower *B*, so the crew needs to go 10 km east and 5 km north. Those instructions can be sent as the direction pair [+10, +5]. The first number gives the horizontal component of the direction, and the second number gives the vertical component.



Reaching All Learners

Act It Out

Place numbered strips on the floor or in the schoolyard to form axes. Give student pairs one card with a coordinate pair and one card with a direction pair. Students find the location designated by the coordinates. One student stands there. The second student finds a position in the direction indicated by the direction pair. The students write the direction pair from this location back to the original location (opposite of the original direction pair). Repeat with different cards.

English Language Learners

Discuss the context of directing firefighters with your English language learners.

Hints and Comments

Overview

Students are introduced to a third method of giving directions. There are no problems on this page for students to solve.

About the Mathematics

In the previous section, students were introduced to coordinates. If the point from which you are looking is the origin of a coordinate system, the coordinates of a point indicate the direction in which you should look.

In this section, students are introduced to direction pairs that indicate a direction from any starting point.

Direction pairs can be used to describe directions from any point, not just from the origin. Both positive and negative numbers are used in the direction pairs. The sign, also the plus sign for positive directions, is always written in the direction pair. The direction pair notation with brackets like this [,]is specific for this unit. Direction pairs are, mathematically speaking, vectors.

A direction pair that describes the direction to a point in the coordinate system as looked at from the origin can be the same as the coordinates of that point. However, the coordinates that give the exact location of the point need not be the same as the direction pair.

B Directions as Pairs of Numbers

Notes

This distinction between parentheses and brackets is important! Just as the x-coordinate is first in a coordinate pair, the x (horizontal) move is first in the direction pair.

4 and **5** It may help some students to draw the lines in the indicated directions first.

5 The starting point is not given. Many will start at the origin, but students may choose any starting point.

B Directions as Pairs of Numbers

Note that direction pairs are in brackets like this: [,]. Coordinates of a point are in parentheses like this: (,).

1. **a.** Write a direction pair to describe the direction of the fire at point *S* as seen from point *A*.

b. Do the same to describe point *S* as seen from point *C*.

2. Using the top half of **Student Activity Sheet 4**, locate and label point *G* at (20,15). Then use direction pairs to describe the location of *G* as seen from points *A*, *B*, and *C*.

Notice that for the rangers at tower *B*, the direction to point *S* is the same as the direction to point *G*. So we can say that the direction pairs [+10, +5] and [+20, +10] indicate the same direction from point *B*.

3. **a.** Why are they the same?

b. Write three other direction pairs that indicate this same direction from point *B*.

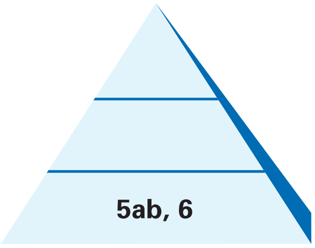
4. Find three different points on the map that are in the same direction from tower *A* as point *S*. Write down the coordinates of these points.

5. **a.** Give two direction pairs that indicate the direction NW.

b. Give two direction pairs that indicate the direction SE.

6. What compass direction is indicated by [+1, 0]? What compass direction is indicated by [0, −1]?

Assessment Pyramid



Write direction pairs and give compass directions.

Reaching All Learners

Intervention

If students have difficulty finding direction pairs, ask them to locate several points that are in the same direction from a tower. Use direction pairs that will move from the tower to the points. Some students will find it helpful to draw the line to identify other direction pairs.

Study Skills

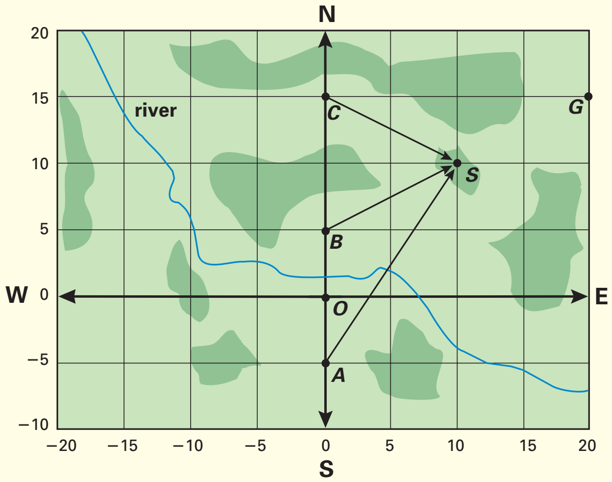
Have students make a Venn diagram or a compare/contrast chart to highlight the similarities and differences between coordinate pairs and direction pairs.

Solutions and Samples

1. a. [+10, +15]

b. [+10, −5]

2.



From point *A*:[+20,+20]

From point *B*: [+20, +10]

From point *C*: [+20, 0]

3. a. Answers will vary. Some students may say that all three points lie on the same straight line from *B*. Other students may indicate that the direction from point *B* to points *S* and *G* is 63°.

b. Answers will vary. One possible set of three directions is: [+40, +20], [+30, +15], [+5, +2.5]. In all directions pairs, the horizontal component should be twice as big as the vertical component.

4. Answers will vary. One possible set of three points is as follows: (2, −2), (4, 1), and (6, 4). The common direction of these points from point *A* is given by [+2, +3], [+4, +6], or [+6, +9].

5. a. Answers will vary. Sample response:

[−5, +5], [−10, +10]

Students' pairs should have a negative first number and a positive second number. The ratio of the two numbers should be equal to −1.

b. Answers will vary. Sample response:

[+5, −5], [+10, −10]

Students' pairs should have a positive first number and a negative second number. The ratio of the two numbers should be equal to −1.

6. [+1, 0] is east

[0, −1] is south

Hints and Comments

Materials

Student Activity Sheet 4 (one per student)

Overview

Students use direction pairs to describe directions. They learn the notation for direction pairs. Students discover that the same direction can be given by more than one direction pair. They also relate the directions described by direction pairs to the wind (or compass) directions that were used in Section A.

Planning

Before students begin to work on problems 1–2, you may want to discuss the map on page 11. Note that brackets ([ ]) are used to denote direction pairs. Students may work on problems 3–6 in small groups. When they are finished, you may want to have a short discussion about these problems.

Comments About the Solutions

1. If students have difficulty, you might have them describe this direction using compass directions first. For example, from point *A* the fire can be seen at 10 km east and 15 km north.

3. Some students may notice that the ratio of the horizontal component of the direction pair to the vertical component is important. Any combination of numbers with the same ratio describes the same direction.

4. Students notice that points that are in the same direction from a given point do not have the same coordinates. Coordinates may only be the same as the direction pair if the direction given is relative to the origin.

5. This is the first time that the starting location for the direction pair is not given. Some students may naturally start at zero, but others might see that these direction pairs describe the correct wind direction from any starting location. This will be made more explicit later in this section. Students should recognize that these two directions are opposite. If students have difficulty seeing this, you might have them compare the direction pairs. The horizontal component and the vertical component in the solution to problem 5b are the opposite of those in the solution to problem 5a.

B Directions as Pairs of Numbers

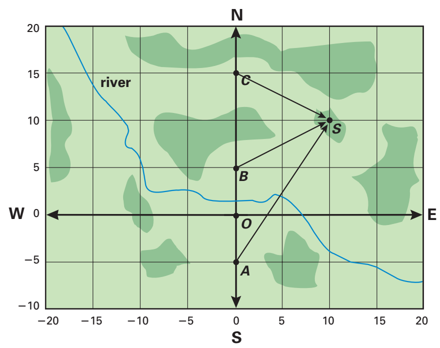
Notes

7 It will be easier to locate the point of intersection (the fire) if students draw the lines in the indicated directions.

10 Some students may select a point (like the origin) from which to begin. Others may just use the ratios of the numbers or the steepness of the steps to compare the directions. Students might also use a ratio table to generate more direction pairs.

11b Hopefully students have observed that direction pairs in the same direction are in the same ratio. They may also observe that the direction pairs can designate points on the same line.

11 and **12** Discuss problem 11 and problem 12 on the next page as a whole class. Look for multiple strategies for 14.



Use the graph on the top half of **Student Activity Sheet 4** for problems 7 through 9.

7. Locate the fire based on the following reports.

- Rangers at tower *B* observe smoke in the direction [−9, +2].
- Rangers at tower *C* observe smoke in the direction [−3, −1].

8. Do the direction pairs [−6, +9] and [−8, +12] indicate the same direction? Use a drawing as part of your answer.

9. a. Locate and label four points that are in the direction [+1, +1.5] from point *A*.

b. What is a quick way to draw all the points that are in the direction [+1, +1.5] from *A*?

10. For each two direction pairs below, explain why they indicate the same direction or different directions.

a. [+1, +3] and [+4, +12]

b. [−4, +3] and [+8, −6]

c. [+5, +8] and [+6, +9]

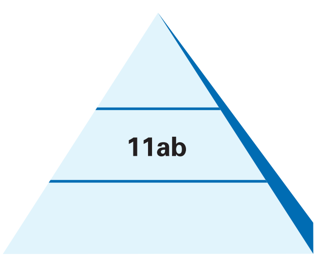
You can use many direction pairs to indicate a particular direction.

11. a. Give five direction pairs that indicate the direction [+12, +15].

b. What do all your answers to part **a** have in common?

c. Could any of the direction pairs you listed have fractions as components? Why or why not?

Assessment Pyramid



Understanding slope in different contexts.

Reaching All Learners

Intervention

Students who have not observed that the numbers in direction pairs are in the same ratio if they indicate the same direction should test this assertion by actually calculating the ratios in some examples.

Advanced Learners

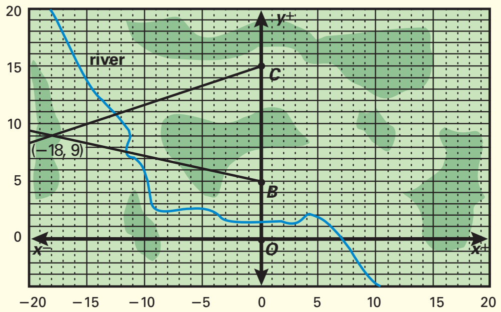
Challenge students to modify the direction pairs given in problem 10c so they will be in the same direction. Find the missing values so [+5, +8], [+1, ?], and [?, +9] will all be in the same direction.

Extension

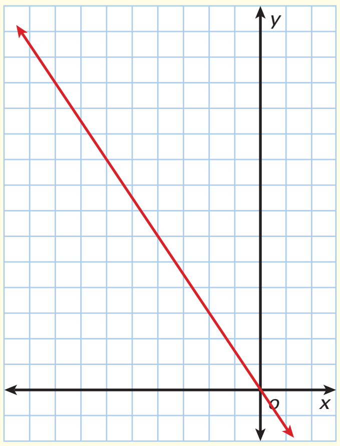
You may want to have students describe the eight main directions of the compass card with direction pairs and then ask them to describe the patterns they see.

Solutions and Samples

7. The fire is located at the intersection point (−18, 9) of the two lines on the following map.



8. Yes, they do indicate the same direction. Some students may verify their answers by drawing the directions starting from the origin as shown on the following graph. Other students may use another starting point.



9. a. Answers will vary. Sample response:

(2, −2), (4, 1), (6, 4), and (8, 7).

b. Connect point *A* with one of the points in problem 9a and extend it.

10. a. Same direction. Going right 1 and up 3 is the same direction as going right 4 and up 12.

b. Different direction. The first pair goes to the left and the second pair goes to the right.

c. Different direction. The direction of the first pair [+5, +8] is along a line that would be slightly steeper than the direction of the second pair.

11. a. Answers will vary. Sample response:

[+4, +5], [+8, +10], [+24, +30], and [+16, +20]

b. Answers may vary. Sample response:

The ratio of the first number to the second (4:5) is the same in each answer.

c. Each component of a direction can also be a fraction. If fractions are used, the ratio of the first number to the second must still be 4 to 5.

Hints and Comments

Materials

Student Activity Sheet 4 (one per student); graph paper (one sheet per student)

Overview

Students investigate direction pairs that are the same and that are opposites. They discover that direction pairs that are opposites form one line.

About the Mathematics

Mathematicians often prefer to simplify the direction pair. Students do not need to do this simplifying, but they may appreciate the mathematician's preference. The concept of slope is informally explored. Directions are the same when the ratios of the numbers in the direction pairs are the same and the signs of the components match. There are many different direction pairs that can describe the same direction. To describe the same direction, the ratios of the numbers in the direction pairs must be the same, and the sign (positive, negative) of the ratio must be the same. The ratios of direction pairs that describe opposite directions are the same. The two line segments in one direction and the opposite direction together form a complete line.

Planning

Students may work on problems 7–10 individually. Problem 10 can be used as an informal assessment.

Comments About the Solutions

8. The graph in the Solutions column shows the direction from the origin. Students may use any other starting point in their drawing.

11. In this problem students may discover that the ratios of the numbers in direction pairs that indicate the same direction are the same. If this does not happen here, there is no problem since this will be explicitly addressed on the next page.

B Directions as Pairs of Numbers

Notes

14 If students have trouble, ask about the ratio of the numbers in the direction pair. Can they find other number pairs with the same ratio? It may help some students to draw the situation on grid paper and use the smaller direction pairs.

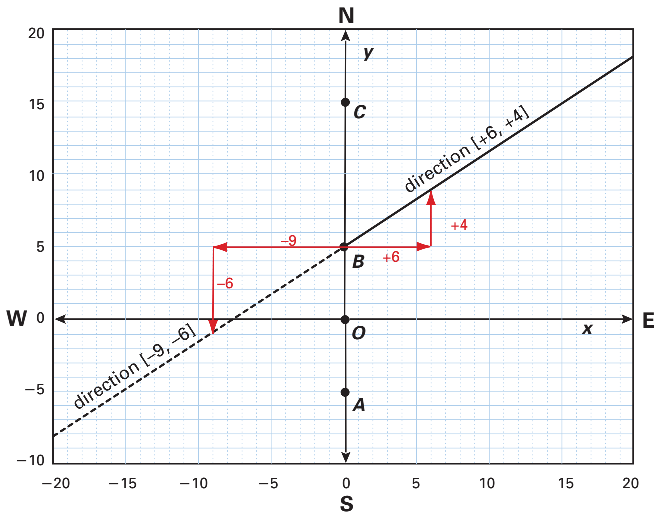
12. Use the map on the bottom of **Student Activity Sheet 4**.

a. Label the point *A* (0, −5) on the map.

b. Show all the points on the map that are in the direction [−1, +2] from *A*.

c. Show all the points on the map that are in the direction [+1, −2] from *A*.

d. What do you notice in your answers for parts **b** and **c**?



The two number pairs [+6, +4] and [–9, −6] represent opposite directions. All the points from B in the directions [+6, +4] and [–9, −6] are drawn in the diagram. The result is a line.

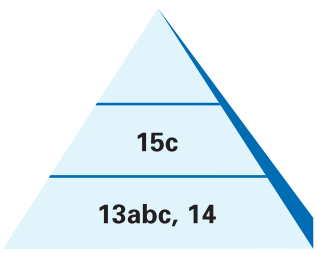
13. a. Give three other direction pairs on the solid part of the line through *B*.

b. Give three other direction pairs on the dotted part of the line through *B*.

c. What do all six direction pairs have in common?

14. Suppose you want to graph the line that has direction pair [+75, +25] and that starts at (0, 10). Describe how you might do this.

Assessment Pyramid



Understand slope;
Identify direction pairs.

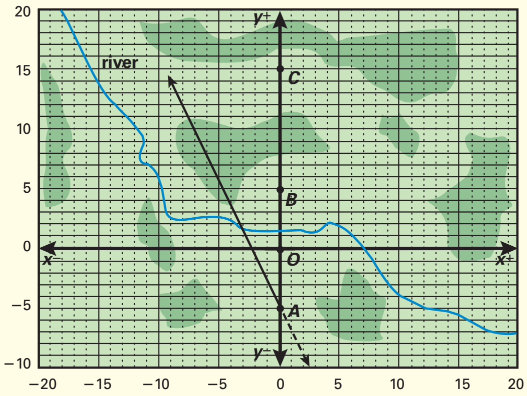
Reaching All Learners

Intervention

Students are used to starting with the horizontal component in coordinates and direction pairs. Students need to switch gears to start with the vertical component in slope. It helps to relate the ratio of the numbers in the direction pair with slope as a measure of steepness—the larger the ratio, the steeper the line. Steepness is a measure of how much a line is going up (or down) in a given distance, so we start with the vertical component.

Solutions and Samples

12. a.–c.



b. Points in direction [−1, +2] are indicated with the solid half-line.

c. Points in the direction [+1, −2] are indicated by the dotted half-line.

d. They form a complete line.

13 a. Answers will vary. Sample response:

[+3, +2], [+12, +8], [+9, +6]

b. Answers will vary. Sample response:

[−3, −2], [−12, −8], [−6, −4]

c. The ratio of the numbers (3:2) is the same in each direction pair. Otherwise stated: the second number is always $\frac{2}{3}$ of the first number, or: the first number is 1 and a half times the second number

14. Answers will vary. Sample answer: Make a coordinate grid, but make the axes using a scale of 10s instead of 1s. You also know that the line with the direction pair [+3, +1] will be the same line because the direction pair has the same ratio as [+75, +25]. So you could plot the direction pair [+3, +1] starting at (0, 10) and extend the line. This will make the line asked for.

Hints and Comments

Materials

Student Activity Sheet 4 (one per student); rulers (one per student)

Overview

Students are introduced to slope as a measure to describe the direction or steepness of a line. They use slope to reason about equivalent fractions.

About the Mathematics

In the unit *Comparing Quantities* students worked with the concept of fair exchange. The principle of fair exchange also may lead to the concept of slope if a graph is used. In the unit *Looking at an Angle,* the concept of slope will be dealt with in a geometrical context, as the tangent of the angle that a line makes with the horizon.

Slope is defined as the ratio of the vertical component of a direction pair to the horizontal component. For different direction pairs this may seem to generate different ratios to indicate the same direction. Simplifying the ratios (fractions) makes clear that the slope is always the same no matter what direction pair is used. Reversely students have a way to understand that different ratios or fractions are actually the same (or opposite) rational number because they indicate the same direction (or slope or steepness).

Comments About the Solutions

12. All points that lie in the same direction and its opposite direction are on one line.

14. Students must use the fact that they can find another direction pair with smaller numbers for the same direction. If students say, "Draw the 75 to the right and 25 up," ask them what they would do if this wouldn't fit the grid.

B Directions as Pairs of Numbers

Notes

15 If students have difficulty finding the slope, ask them to find two points on the line. Find the vertical change and the horizontal change between the points. How does this relate to the direction pair?

Read About the Mathematics to see the connection between this definition of slope and the more familiar one $m = \frac{\triangle y}{\triangle x}$

Up and Down the Slope

All the number pairs for a single direction and for the opposite of that direction have something in common: they all have the same ratio.

You can calculate two different ratios for a number pair:

horizontal component divided by vertical component

or

vertical component divided by horizontal component

Mathematicians frequently use this ratio: $\frac{\textbf{vertical component}}{\textbf{horizontal component}}$

and call that ratio the **slope** of a line. **slope** = $\frac{\textbf{vertical component}}{\textbf{horizontal component}}$

15. **a.** Find the slope of the line you drew in problem 12, using the direction [−1, +2] given in 12b.

b. Do the same as in part **a,** but now use the direction [+1, −2] from 12c.

c. **Reflect** What do you notice if you compare your answers to problems 15a and 15b?

From problem 13, you can conclude that $\frac{4}{6} = \frac{-6}{-9}$.

16. **a.** Explain how you can conclude this from problem 13.

b. Using direction pairs, explain that $\frac{-4}{2} = -2$.

Reaching All Learners

Intervention

If students have problems, refer them to problems 15 and 16a where the same type of reasoning is used.

Extension

You may want to provide students with more lines and ask them to find the slopes of the lines.

Vocabulary Building

Have students add the term *slope* to the vocabulary section of their notebooks. Make sure they give examples.

Solutions and Samples

15. a. Slope is $\frac{+2}{-1} = -2$.

b. Slope is $\frac{-2}{+1} = -2$.

c. The slopes are the same.

16. a. Sample answer: The direction pairs [+6, +4] and [−9, −6] describe the same line. This is shown in problem 13. A line has only one slope, so both ratios vertical/horizontal $\frac{+4}{+6}$ and $\frac{-6}{-9}$ are the same.

b. Sample answer: The slope $\frac{-4}{2}$ is made from the direction pair [+2, −4] but can also be made from the pair [+1, −2], the slope is $\frac{-2}{+1} = -2$.

Hints and Comments

Materials

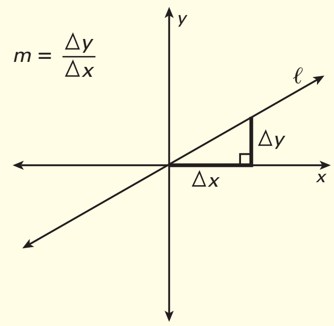
rulers (one per student);
graph paper (optional, one sheet per student)

Overview

Students solve problems about slope. They find the slopes of lines, and draw lines with a given slope.

About the Mathematics

The slope of a line may be found by drawing any convenient right triangle using the line as the hypotenuse and then finding the ratio of the vertical component to the horizontal component.



Students may still want to write a direction pair first and use this to find the ratio of the vertical component to the horizontal component.

It is important to note that this definition for slope, the ratio of the vertical component to the horizontal component, is well-defined. That is, any two points on the same line that are selected to find the vertical and horizontal components will result in the same slope.

This definition for slope also leads to the two-point form for the equation of a line, which is often used in high school algebra courses: $m = (y - y_1)/(x - x_1)$ so $y - y_1 = m(x - x_1)$

Planning

Before students begin working on problems 15 and 16, you may want to discuss the concept of slope. Remind students of previous units in which they have seen and worked with slope.

Comments About the Solutions

15. c. Opposite direction pairs are on the same line, so the slope is the same. Finding the slope from opposite direction pairs can also be related to integer division.

16. b. In this problem, students must decide themselves which direction pairs they want to use to clarify the equation.

B Directions as Pairs of Numbers

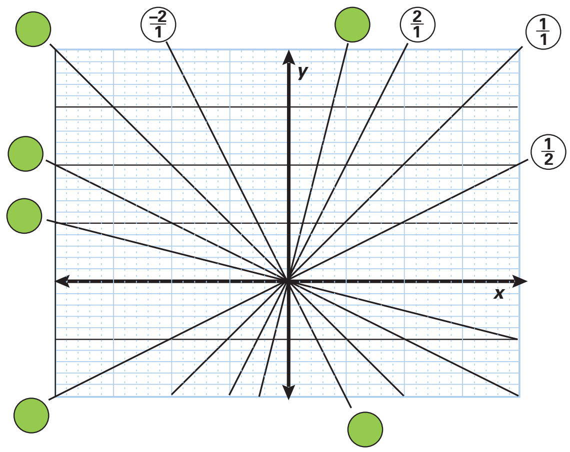
Notes

Point out that the slope 2 is a ratio: $\frac{2}{1}$.

Encourage students to write slopes as fractions to reinforce this idea.

17a It is helpful to find the points where the line crosses the corner of a grid square. Students should find several of these points for each line. Students can draw the vertical and horizontal move from one point to the next.

17b Discuss the multiple strategies students use when solving this problem.



Use **Student Activity Sheet 5** for problems 17 through 19.

Each of the lines drawn on the **coordinate grid** contains the point (0, 0). For some of the lines, the slope is labeled inside its corresponding circle.

17. a Fill in the empty circles with the correct slope.

b. What is the slope for a line that goes through the points (1, 1) and (15, 3)? How did you find out?

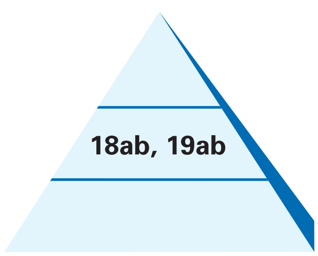
18. a. What do you know about two lines that have the same slope?

b. Explain that $\frac{3}{1}$, $\frac{6}{2}$, $\frac{-3}{-1}$, and $\frac{15}{5}$ all indicate the same slope. What is the simplest way to write this slope?

19. Draw and label the line through (0, 0) whose slope is:

a. $\frac{4}{3}$ **b.** $-\frac{1}{2}$

Assessment Pyramid



Understand slope.

Find the point of intersection of two lines.

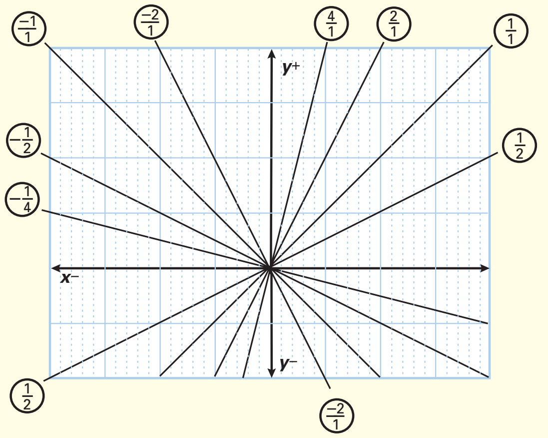
Reaching All Learners

Intervention

If students have problems, they may want to draw the two points, connect them, find a direction pair, and calculate the slope. This can be done either in a coordinate grid or with a rough sketch.

Solutions and Samples

17. a.



17. b. The slope is $\frac{2}{14} = \frac{1}{7}$ Students' explanations will vary. Sample explanation:

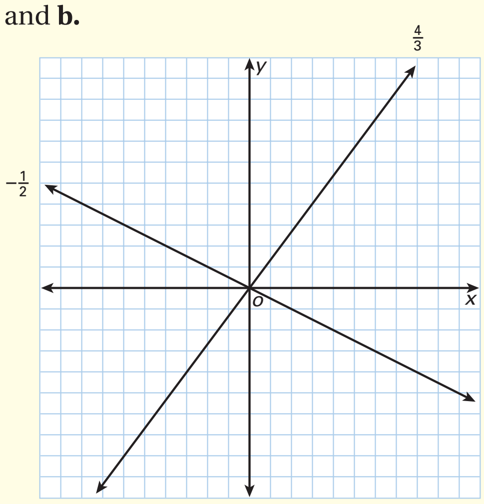
The direction from (1, 1) to (15, 3) is 14 steps to the right and two steps up. This can be written as the direction pair [+14, +2]; the slope can be found using the rule from Student Book page 15.

18. a. They either form one complete line if they also contain the same point like in the drawing for problem 17; otherwise they are parallel.

b. Sample answer: Directions pairs for these slopes are:[+1, +3], [+2, +6], [−1, −3], [+5, +15]. If you draw all these directions starting in the same point, all endpoints lie on the same line, so all ratios or slopes are the same.

This can also be seen because all ratios can be simplified to the same number; this is 3, and this is the simplest way to write this slope.

19. a. and **b.**



Hints and Comments

Materials

Student Activity Sheet 5 (one per student); transparency of **Student Activity Sheet 5** (optional); graph paper (optional, one sheet per student)

Overview

Students find the slopes of lines, and draw lines with a given slope.

Comments About the Solutions

17. a. Students may want to write a direction pair for each line first and use this to calculate the slope. You may ask students to look for patterns in the slopes they found (like positive-negative, inverse) and discuss these.

19. If students have difficulty, ask them to write a direction pair fitting the given slope first.

B Directions as Pairs of Numbers

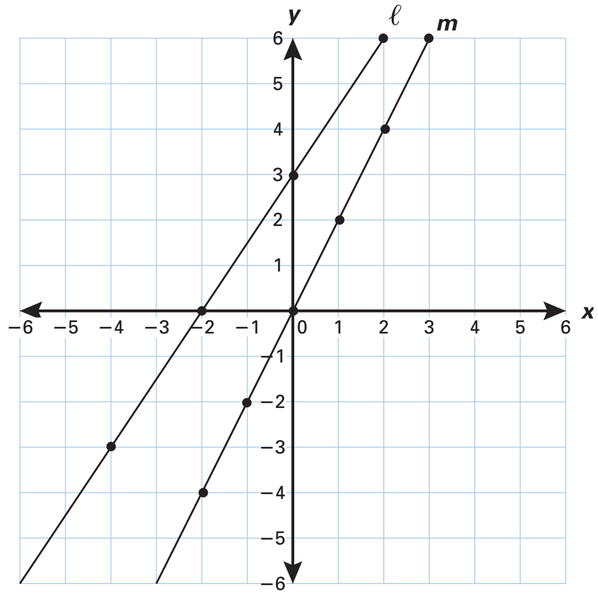
Notes

20 This problem can generate lots of interesting strategies. Take time to have students present their solutions. You may want copies of the problem on overhead transparencies to facilitate this.
Note: Students need to keep their work from this problem to use in Section E.

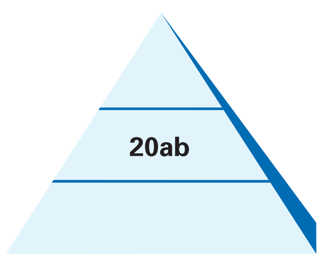
The two lines in the graph below are not parallel.

20. a. Find the slope of each line.

b. This grid is too small to show the point where the two lines meet. Find the coordinates of this point and explain your method for finding it.



Assessment Pyramid



Understand slope.

Find the point of intersection of two lines.

Reaching All Learners

Extension

As students work on problem 20, you may want to have them discuss the connection between slope and parallel lines.

Solutions and Samples

20. a. The line (m) passing through the origin has a slope of 2. The line l on the left has a slope $\frac{3}{2}$.

b. The coordinates are (6, 12). Students may use a variety of strategies to solve this problem. Sample strategies:

Strategy 1

Some students may reason using the patterns in the vertical and horizontal distances between the two lines.

At $x = -2$, the vertical distance between the lines is 4.

At $x = 0$, the vertical distance between the lines is 3.

At $x = 2$, the vertical distance between the lines is 2.

At $x = 4$, the vertical distance between the lines is 1.

At $x = 6$, the vertical distance between the lines is 0.

At $y = 0$, the horizontal distance between the lines is 2.

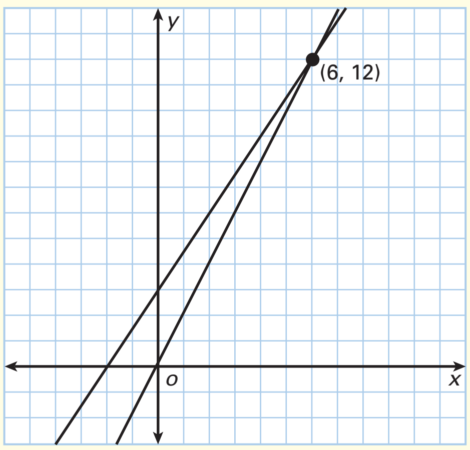
At $y = 6$, the horizontal distance between the lines is 1.

At $y = 12$, the horizontal distance between the lines is 0.

So at (6, 12), the distance is 0 for both x and y.

Strategy 2

Some students may redraw the two lines on a sheet of graph paper, extend the lines, find the place where they meet, and estimate the intersection point.



Hints and Comments

Overview

Students solve problems about slope. They use slope to locate a point of intersection for two lines.

Comments About the Solutions

20. b. Students are not expected to solve a system of equations to find the point of intersection of these two lines. Encourage them to use an intuitive, informal strategy.

B Directions as Pairs of Numbers

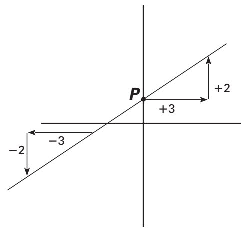
Notes

Read the Summary aloud as a class. Ask students how the Summary might be helpful.

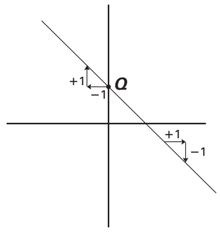
B Directions as Pairs of Numbers

Summary

You can indicate a direction from a point, using a direction pair such as [+3, +2] or [+1, −1]. The first number is the horizontal component, and the second number is the vertical component.



From *P*, the points in the directions [+3, +2] and [−3, −2] are on the same line. The slope of this line is $\frac{2}{3}$.



From *Q*, the points in the directions [+1, −1] and [−1, +1] are on the same line. The slope of this line is $\frac{+1}{-1} = -1$.

Brackets are used to distinguish direction pairs from coordinate pairs.

[+2, −4] is a direction pair.

(2, −4) are the coordinates of a point.

All direction pairs in the same and opposite direction have the same ratio.

The slope of a line is given by this ratio:

$$\textbf{slope} = \frac{\textbf{vertical component}}{\textbf{horizontal component}}$$

If you want to draw a line whose slope is given, you may want to find a direction pair first that fits the given slope.

Reaching All Learners

Study Skills

Before looking at the Summary, ask students to review Section B and write down what they think are the important ideas in the section.
After discussing this in small groups, ask students to look at the Summary. What topics from the Summary were included on their lists? Does the Summary include items not on their lists? Do they have items that were not in the Summary? This strategy helps students review, helps them develop the ability to identify important ideas, ensures that they actually read the Summary, and helps them appreciate the value of the Summary section.

Hints and Comments

Overview

Students read the Summary, which reviews the main concepts covered in this section.

B Directions as Pairs of Numbers

Notes

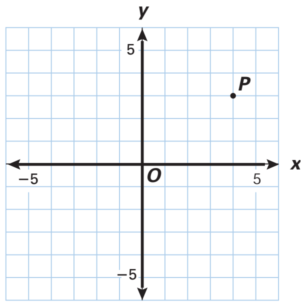
1 Check that students are putting the *x* value first in the pair and that they are using parentheses and brackets correctly.

1d and **2** These are good discussion questions. Look for a variety of student approaches.

Check Your Work

1. a. Give the coordinates of point *P* in this coordinate system.

b. Give two direction pairs that describe the direction from *O* to point *P* in the coordinate system.



c. Copy the drawing in your notebook. Locate and label three points that are in the direction [−4, −2] from point *P*.

d. What is a quick way to draw all points in the direction [−4, −2] from point *P*?

2. For each two direction pairs below, say whether they indicate the same or different directions and explain why.

a. [+4, +3] and [+8, −6]

b. [+5, +8] and [+1, +1.6]

c. [+13, 0] and [+25, 0]

d. [+0.5, +2] and [+2, +8]

3. a. Draw a coordinate system in your notebook like the one for problem 1; mark point *P* from problem 1 in the grid you drew. Mark point *Q* with coordinates (1, 1).

b. What direction pair describes the direction from *P* to *Q*?

c. Draw the line through *P* and *Q* and find its slope.

Reaching All Learners

Accommodation

It may be easier for students to use grid paper rather than notebook paper for 1c and 3. Some students may wish to use grid paper for 2 if they use a drawing strategy.

Parent Involvement

Have parents review the section with their child to relate the Check Your Work problems to the problems from the section.

Solutions and Samples

Answers to Check Your Work

1. a. P (4, 3)

   b. The direction from O to P can be described with the direction pair [+4, +3] and the direction pair [+8, +6] or [+2, +1.5] or other pairs that have the same ratio as $\frac{+3}{+4}$.

   c. Different points can be labeled, for example, (−4, −1) and (−2, 0) and (2, 2). Note that all points must lie on a straight line through P and (−2, 0).

   d. You can draw a line through P and any of the points mentioned in your answer to 1c.

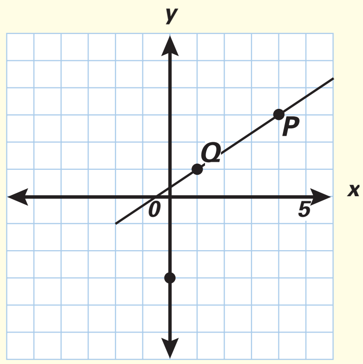
2. a. Different direction. The first pair moves right and up, and the second pair moves right and down.

   b. Same direction. They both go to the right and up.

   c. Same direction. They both go due east.

   d. Same direction. They both go right and up.

3. a. See graph below.



   b. The direction from P to Q can be described with the direction pair [−3, −2].

   c. The slope can be found by using the direction pair from part **b**. So the slope is $\frac{-2}{-3}$, which can be simplified to $\frac{2}{3}$.

Hints and Comments

Overview

Students work on the Check Your Work problems about direction pairs and slope. These problems are designed for student self-assessment. A student who can answer the questions correctly has understood enough of the concepts taught in the section to be able to start the next section. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work.

Answers are provided in the Student Book. Have students discuss their answers with classmates.

Directions as Pairs of Numbers

Notes

5a If students have trouble understanding this problem, have them draw two points. How many ways can they connect them with a straight line?

5b It is possible to get the correct slope using incorrect reasoning! Check that the students use a strategy that looks at the ratio of vertical change to horizontal change between the two points. If students look at the ratio between the x- and y-coordinates of the same point, they will see the same ratio. This is true because the line goes through the origin. Not every line going through the point (1, 2) has a slope of 2!

For Further Reflection

Reflective questions are meant to summarize and discuss important concepts.

Directions as Pairs of Numbers

4. In the coordinate system you drew for problem 3, draw and label the line m through O (0, 0) that has a slope of 2.

5. a. How many lines contain both points (1, 2) and (26, 52)? Explain your reasoning.

b. Find the slope of the line(s) in part **a**. How did you find it?

For Further Reflection

How can similar triangles be used to find the slope of a line?

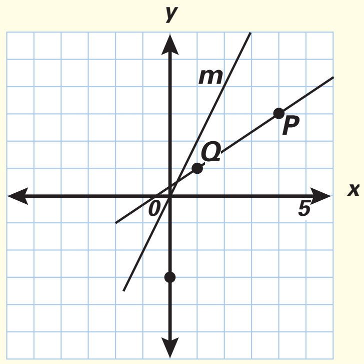
Reaching All Learners

Hands-On Learning

Slope provides a nice opportunity for a "field trip." Provide students with rulers, meter sticks, and/or measuring tapes and have them find the slope of stairs, banisters, access ramps, and/or hillsides near school. Ask students to plan what measurements they need to take before leaving the classroom. Note: All measurements for the same slope need to be in the same units.

Solutions and Samples

4. See graph below.



5. a. One line goes through these two points. No more than one line can be drawn through the same two points. (You can try to draw more than one line however it will still be the same line).

b. To find the slope, you must first find the direction from one point to the other. From (1, 2) to (26, 52), you go 25 steps in a horizontal direction and 50 steps in a vertical direction. So the direction pair is [+25, +50]. The slope is $\frac{+50}{+25} = 2$. It may help you to make a sketch of the situation.

For Further Reflection

This is a challenging question. Many students may not make the connection between similar triangles and finding the slope of a line. This may be worth exploring in class discussion after students have been given time to develop their own response.

Sample response:

You can use similar right triangles to find the slope of a line. If you compare the side lengths of each triangle, each pair of sides should be the same ratio. The ratio of the vertical side to the horizontal side for these similar triangles should have the same slope, for example, $\frac{1}{3}$, $\frac{2}{6}$, and so on.).

Hints and Comments

Overview

Students work on the Check Your Work and For Further Reflection problems about direction pairs and slope.

Check Your Work Problems

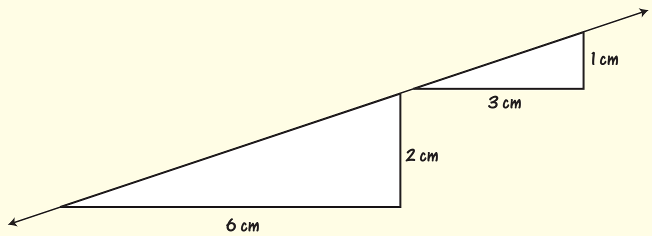
These problems are designed for student self-assessment. A student who can answer the questions correctly has understood enough of the concepts taught in the section to be able to start the next section. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work.

Answers are provided in the Student Book.
Have students discuss their answers with classmates.

Comments About the Solutions

For Further Reflection

Encourage students to include both numerical and descriptive information in their responses.



Section Focus

Students use the direction of a line (described with a number pair) to explore how a line is drawn by taking bigger or smaller steps. This leads to a rule for finding the y-value of a point on a line after a certain number of horizontal steps. This rule leads to a formula for the equation of a line, with a starting point (later the y-intercept) and a slope. Students interpret the meanings of numbers in equations, and draw the lines described by equations. They describe the steepness of a line by measuring the angle the line makes with the x-axis. Students investigate when the tangent of that angle is the same as the slope of the line.

Pacing and Planning

Day 8: Directions and Steps		**Student pages 21 and 22**
INTRODUCTION	Problems 1 and 2	Investigate the use of direction pairs to draw a line on a computer screen.
CLASSWORK	Problems 2–5	Use horizontal and vertical steps to informally investigate the equation of a line in slope-intercept form.

Day 9: Directions and Steps (Continued)		**Student pages 22 and 23**
INTRODUCTION	Problems 6–8	Introduce and investigate the equation of a line and y-intercept.
CLASSWORK	Problems 9–11	Graph lines using the slope and y-intercept.
HOMEWORK	Problem 12	Write equations for six graphed lines and investigate the equations of parallel lines.

Day 10: What's the Angle?		**Student pages 24 and 25**
INTRODUCTION	Problems 13–15	Investigate the relationship between the slope of a line and the angle that the line makes with the positive x-axis.
CLASSWORK	Problems 16–19	Introduce the relationship between the slope and the tangent of the angle that a line makes with the positive x-axis.
ASSESSMENT	Check Your Work For Further Reflection	Student self-assessment: Section C Goals

Additional Resources: *Algebra Tools*; Additional Practice, Section C, page 46

Materials

Student Resources
No resources required.

Teachers Resources
No resources required.

Student Materials
Quantities listed are per student.

- Compass card or protractor
- Graph paper
- Ruler

* See Hints and Comments for optional materials.

Learning Lines

Directions and Steps

In the previous section, students have investigated the use of direction pairs to describe directions. They have seen that different pairs may describe the same direction and that all points that lie in a given direction or its opposite starting from the same point form a line. Now students investigate how you can move along a line by taking steps in a certain direction. Taking horizontal steps of +1 leads to a rule to find the coordinates of points on the line, for example:

Starting point: (0, 5).

After 100 horizontal steps of +1:

$x = 100$

$y = 5 + 100 \text{ x } 2 = 205$

These rules lead to a formula for a line relating the x-coordinates and the y-coordinates. For the example above, this formula is: $y = 5 + 2x$.

Equation of a Line

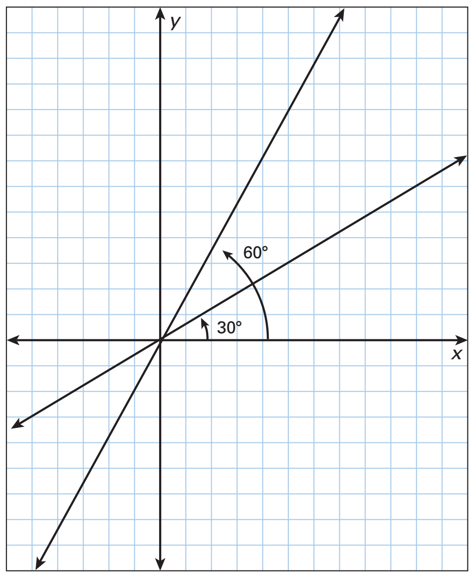
In this section, equations for lines are introduced. Students have seen equations of lines in other *Mathematics in Context* units. But these were mostly equations using word variables that relate to the context. Here the equations are more formally introduced and written in y and x.
A common way to write the equation of a line is in the form: $y = mx + b$, where the variable m is the slope and the variable b is the y-intercept.

Starting from steps along a line, students investigate the role of the numbers in a rule for finding coordinates. They do the same for the numbers in the resulting equation by relating the equation to the rule and to the graph. In this way they learn about the slope (see also Section B) and the y-intercept.

The y-intercept, is where a line crosses the y-axis. This concept is formally introduced in this unit. Algebraically, the y-value of the equation equals the y-intercept when the x-value is 0. The slope indicates how steep a line is. Students draw lines for given equations and write equations for drawn lines.

Slope and Tangent

In this section, students investigate if the tangent of the angle that a line makes with the positive (or right) side of the x-axis is equal to the slope. This is the case only if both the horizontal and the vertical axes are scaled in the same way. The tangent of an angle is defined as the vertical distance divided by the horizontal distance. Students discover that the slope is not proportional to the angle.



At the End of This Section: Learning Outcomes

In this section, students develop many skills: At the end, they can find the equation of a line from the graph of a line, graph a line given its equation, and investigate angles related to slope.

They can use and find equations of lines in the coordinate plane in the form: $y = mx + b$, and they understand the meaning of slope and y-intercept.

Notes

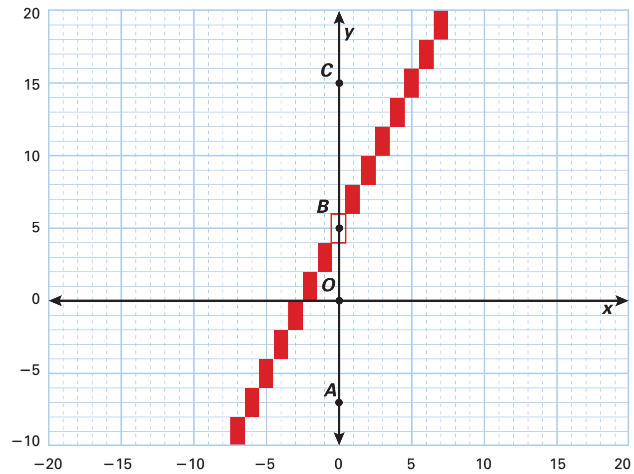
1a and **b** Do this problem as a whole class, or discuss questions 1 and 2 after students have worked on them. This will help all students with the questions that follow.

1b Students might complete the sequence 5, 7, 9, . . . 25. This will be more difficult for 1,000 steps. Students will have to think of a formula or pattern. One possible pattern is as follows: Since you add two vertically for each step, after 1,000 steps you have added 2 × 1,000 = 2,000. Then you have to add the five you started with to arrive at 2,005.

C An Equation of a Line

Directions and Steps

In the coordinate system below, a line is drawn in the direction [+1, +2] from *B*.



You can think of moving along this line one step at a time. Each step is a move of +1 unit horizontally and +2 units vertically.

(0, 5)
+1 ⇩ ⇩ +2
(1, 7)
+1 ⇩ ⇩ +2
(2, 9) etc.

1. **a.** The description shows two steps along the line. Where are you after 10 steps?

b. Where are you after 25 steps? After 1,000 steps?

This description shows steps along the same line but in the opposite direction.

(0, 5)
−1 ⇩ ⇩ −2
(−1, 3)
−1 ⇩ ⇩ −2
(−2, 1) etc.

2. **a.** Where are you after 10 steps?

b. After 100 steps?

Reaching All Learners

Extension

Using a computer or GC projector, you may want to demonstrate how to use a graphing calculator or a computer program to draw the line shown on page 21 of the Student Book. The equation of this line is $y = 5 + 2x$.

Have students examine exactly how the graphing calculator or computer forms a line. Rather than drawing a straight line, the graphing calculator or computer actually turns on a series of pixels that lie along a straight line. Such a line often looks jagged (sometimes it is smooth). Whether the points can be identified or not depends on the resolution of the screen. You might have students consider whether there is a relationship between the slope of a graphed line and the way the pixels are presented on the screen.

Solutions and Samples

1. a. You are at (10, 25) after 10 steps.
   b. You are at (25, 55) after 25 steps and (1,000, 2,005) after 1,000 steps.
2. a. $(-10, -15)$
   b. $(-100, -195)$

Hints and Comments

Overview

Students investigate how you can move along a straight line by taking steps in horizontal and vertical directions.

About the Mathematics

A straight line is actually a collection of an infinite number of points. Theoretically, the points themselves have no dimensions, and the line has no thickness. In practice, however, a line does have a thickness, and a point does have dimensions.

Comments About the Solutions

2. This problem is related to problem 1. If students have difficulties, refer them back to problem 1.

C An Equation of a Line

Notes

3 Some students may find it easier to move one step at a time; others will see the pattern and move directly to the endpoint. Some students have difficulty with decimal computation; the practice can be helpful.

4a Ask about the equation above 4a, *Why does $5 + 100 \times 2 = 205$ and not 210?* This is a good place to review order of operations.

4 Students need to pay attention to order of operations when doing the calculations.

5b Negative values of x represent steps to the left on the line.

A computer or graphing calculator can quickly calculate and draw all of the points on a line. Suppose a computer takes horizontal steps of +0.1 and −0.1 when drawing the points on this line.

3. a. What are the corresponding vertical distances for each step the computer takes?

b. If you start at (0, 5), where are you after 8 steps when +0.1 is the horizontal distance?

c. If you start at (0, 5), where are you after 3 steps when −0.1 is the horizontal distance?

Here is a rule you may have discovered.

Starting point: (0, 5).
After 100 horizontal steps of +1:
$x = 100$
$y = 5 + 100 \times 2 = 205$

4. a. Explain what each of the numbers in $y = 5 + 100 \times 2 = 205$ refers to.

b. Write a similar rule for 75 horizontal steps of +1.

c. Write a rule for 175 horizontal steps of +1.

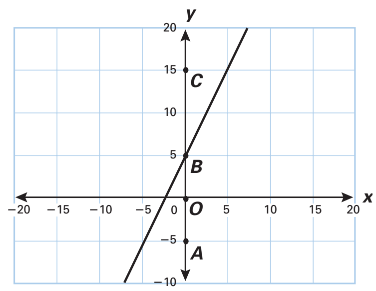
d. Write a rule for $3\frac{1}{2}$ horizontal steps of +1.

From the rules you wrote in problem 4, you can find a formula relating the x-coordinates and the y-coordinates:

$y = 5 + x \cdot 2$ or $y = 5 + 2x$

5. a. Explain the formula.

b. Does the formula work for negative values of x?



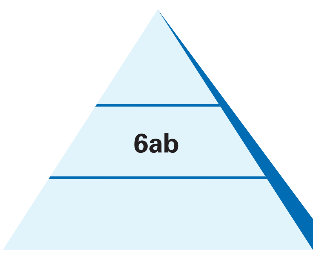
The formula $y = 5 + 2x$ is called an **equation of a line**. If you draw a graph for this equation, you see a line like this.

In the equation $y = 5 + 2x$, two numbers play special roles.

6. a. What is the importance of the "5" for the graph?

b. What is the importance of the "2" for the graph?

Assessment Pyramid



Understand the graph of a line in the coordinate plane.

Reaching All Learners

Intervention

If students have difficulty with problem 4, have them carefully analyze the equation in part a. Only the step size changes in the remaining parts.

Parent Involvement

You may want to ask students who have a computer or graphing calculator at home to draw a line, describe what it looks like on the screen, print it, and bring the description and the printout to class. Then you can discuss the descriptions and printouts as a class.

Vocabulary Building

Have students add the term *equation of a line* to the vocabulary section of their notebooks.

Solutions and Samples

3. a. $+0.2$ and -0.2

b. (0.8, 6.6). Strategies will vary. Sample strategy:

Complete eight steps of $+0.1$ starting from ($x =$) 0 and eight steps of $+0.2$ starting from ($y =$) 5.

c. (-0.3, 4.4)

4. a. The 5 is the starting value of y at the starting point (0, 5); the 100 indicates the 100 steps to be taken; the 2 is the vertical distance for each step (if the horizontal distance is $+1$, the corresponding vertical distance in the direction along the line is $+2$); the resulting number 205 is the y-coordinate of the "end-point" on the line.

b. After 75 horizontal steps of $+1$,

$x = 75$

$y = 5 + 75 \text{ x } 2 = 155$.

c. After 175 horizontal steps,

x = 175

y = 5 + 175 x 2 = 355

d. After $3\frac{1}{2}$ horizontal steps of $+1$,

$x = 3.5$

$y = 5 + 3.5 \text{ x } 2 = 12$.

5. a. Explanations will vary. Sample explanations:

- The y-coordinate is equal to two times the x-coordinate plus five.
- The y-coordinate can be found by starting at 5 and adding the number of steps (x) times the length of each vertical step, which is 2.

b. Yes.

6. a. The "5" is where the line crosses the y-axis. Some students may also say that the 5 is the starting number for y.

b. The "2" is the slope. You move two units vertically (the length of a step) for every one unit horizontally.

Hints and Comments

Overview

Students calculate points that are on a line and write rules to describe how particular straight lines are drawn.

About the Mathematics

The rules for a straight line are written as a generalized calculation rule on this page. Students relate the meaning of slope and y-intercept to both the equation and the graph.

Planning

Students may work on problems 3 and 4 in small groups, and 5 and 6 individually.

Comments About the Solutions

3. After students finish problem 3, you may want to discuss their answers and ask students what rule they can formulate based on their calculations.

5. and **6.**
Problems 5 and 6 are critical because students are asked to make a connection between an algebraic rule and the graph of a line. This is prepared in problem 4a.

5. b. This can be understood because the line runs through points with negative coordinates as well. Note: Negative x's indicate steps in a negative (opposite) direction, as was the case in direction pairs. See also problem 3 where the length of the step has the minus sign and not the number of steps.

6. a. Encourage students to describe the meanings of "5" and "2" for the graph in their own words. Discuss which descriptions make sense and why. Refer to the rules students wrote in problem 4.

Extension: Using Technology

You may want to have students use a graphing calculator or computer to enter the equation $y = 5 + 2x$ and graph the line. You can present this problem:

> Use a graphing calculator to show $y = 5 + 2x$. Explain how to change the window of your calculator to display the same range of x-values and y-values as shown in the graph on page 21.

Using the graphing calculator, have students use the trace feature to investigate the actual pixels used to create a line. You might also have them use the table feature along with tblset to generate a table of values that match the trace step.

An Equation of a Line

Notes

7 This question relates the equation of a line to its graph. Using an overhead transparency of the graph in 5b, have students physically show the "5" on the line and connect it to the term *y*-intercept. Do the same with the "2" and the slope.

11 Some students may remember or may figure out that a 45° angle makes a line with a slope of 1. Students can also draw the angle on graph paper and then find the slope of the line.

12c Have students find the value of *y* when $x = 0$ and show the point on the graph. This reinforces the connection between these representations

There are special names for the 5 and the 2 in the equation $y = 5 + 2x$. The 2 is called the *slope*, and the 5 is called the *y-intercept*.

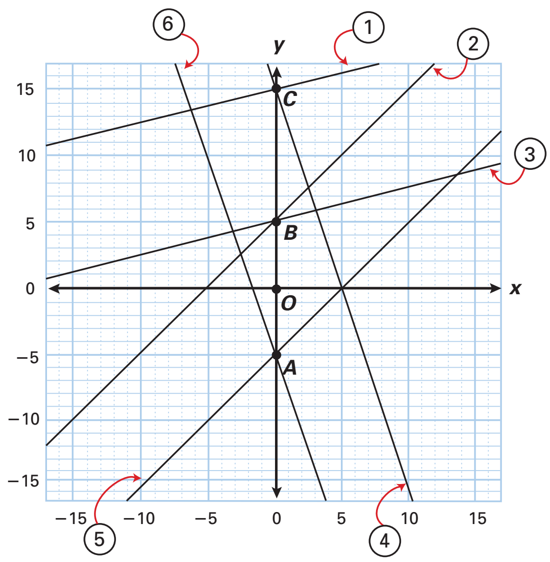
7. Why do you think it is called the *y*-intercept?

8. Using the graph on page 22 write the equation for a line that goes through point *C* and has a slope of 2.

9. Make a copy of the graph shown on page 22 on a piece of graph paper.
   a. Show the line through *B* with slope $\frac{1}{2}$. Then label the line with its equation.
   b. Show the line through *C* with slope $\frac{3}{6}$ and label the line with its equation.
   c. What do you notice about the two lines? Justify your answer.

These two equations represent the same line:

$$y = 5 + (-2) \cdot x \qquad \text{and} \qquad y = 5 - 2x$$

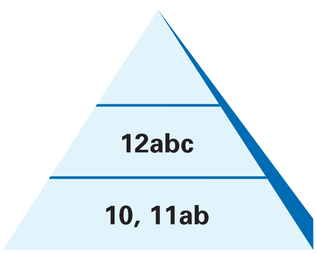


10. Explain why the equations represent the line through *B* with slope −2.

11. a. Write an equation for the line that contains *B* and forms a 45° angle with the direction east.
    b. What is the equation if the line contains *O* instead of *B*?

12. a. In your notebook, write the equation for each of the six lines in the grid to the left.
    b. Which lines are parallel? Explain your answers.
    c. For all equations, find the value of *y* for $x = 0$. What do you notice?

Assessment Pyramid



Understand the graph of a line in the coordinate plane.

Find equations given the slope and *y*-intercept.

Reaching All Learners

Vocabulary Building

Ask students where they have heard the term *intercept*, or *interception*, before. How do these connect to *y*-intercept?

Important vocabulary introduced here includes *equation of a line* and *y-intercept*; and the term *slope* recurs. If students have illustrated words from other sections, they can add illustrations for these.

Solutions and Samples

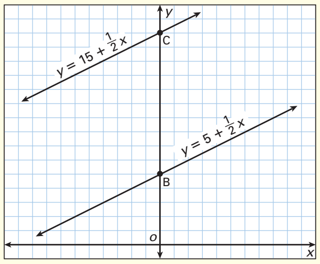
7. The y-intercept is the value of y where the line crosses, or intercepts, the y-axis.

8. $y = 15 + 2x$

9. a. $y = 5 + \frac{1}{2}x$

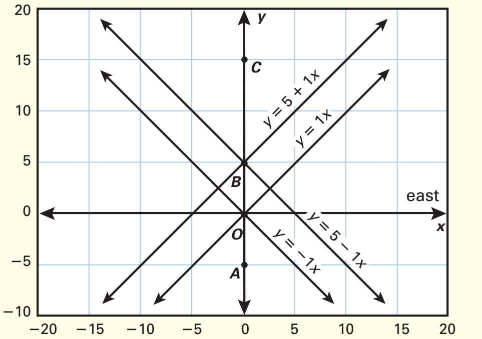
   b. $y = 15 + \frac{1}{2}x$

   c. They are parallel. They have the same slope and different y-intercepts, so they will never meet.



10. The line passes through point B (0, 5). This can be seen from the "5" in the equations. The slope of -2 means that the vertical component is -2 when the horizontal component is $+1$, so the equation of the line is $y = 5 + -2x$. This equation can also be written as $y = 5 - 2x$.

11.



a. $y = 5 + 1x$, or $y = 5 + x$

b. $y = 0 + 1x$ (or: $y = x$). See graph above.

12. a. equation 1: $y = 15 + \frac{1}{4}x$

equation 2: $y = 5 + x$

equation 3: $y = 5 + \frac{4}{15}x$

equation 4: $y = 15 - 3x$

equation 5: $y = -5 + x$

equation 6: $y = -5 - 3x$

b. The following lines are parallel: 2 and 5; 4 and 6. Each pair of lines has the same slope.

c. equation 1: $y = 15$

equation 2: $y = 5$

equation 3: $y = 5$

equation 4: $y = 15$

equation 5: $y = -5$

equation 6: $y = -5$

The y-values for $x = 0$ are the same as the y-intercept. They are the y-coordinates of the points where the cross the y-axis. Lines 1 and 4 have the same y-intercept. They go through point C. Lines 2 and 3 have the same y-intercept. They both go through point B. Lines 5 and 6 have the same y-intercept. They both go through point A.

Hints and Comments

Overview

Students are introduced to the equation of a line (using slope and y-intercept); they investigate this equation and relate it to the graph.

About the Mathematics

There are several ways to find the equation of a line. At this point, it is sufficient for students to use the strategy of finding the y-intercept (from the graph), finding the slope (from the graph, for instance by writing a direction pair for the line first), and writing the equation of the line. Lines are parallel if their slopes are the same.

Planning

Students may work on problems 7–12 individually.

Comments About the Solutions

8. Students should recognize that only the y-intercept changes here.

9. To draw a line, students may want to find a direction pair that fits the slope first.

10. Students need to recognize that adding a negative is the same as subtracting. Some students find it easier to work with the unsimplified form.

11. If students have difficulty finding the equations, have them draw the lines first. You may want to remind students that $1x$ is usually just written as x.

12. Students may have difficulty finding the slope of each of these lines because they need to find points with easy-to-read coordinates. The right ends of lines 1, 2, 3, and 5 each leave the grid at an easy-to-read point. You may want to suggest that students use the coordinates of these points to find the slopes. Here they may also want to write a direction pair first.

12. b. If students use the graph to tell which lines are parallel, ask them if they can be sure and ask how they can use the equations as well.

Extension: Using Technology:

You may have students display the six lines shown in the graph on Student Book page 23 on their graphing calculators. Have students do this problem: Make three lines on a graphing calculator. Show them to a partner. Have the partner write an equation for each line.

An Equation of a Line

Notes

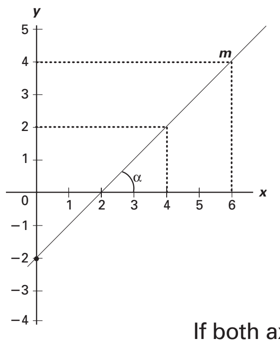
13c Students are now measuring angles with 0° measure from the horizontal. This is not directional, where 0° is north. To separate these two, it helps to use a compass card for direction and a protractor for angles. Be sure students measure angles from the horizontal in the rest of this section.

14b Accept reasonable estimates. These slopes are irrational numbers.

15 Students need to find the slope using one of the previous methods and to measure the angle the line makes with the horizontal to find the tangent. The two values should be similar, but may not be identical, due to measurement error.

What's the Angle?

Line m is drawn in the coordinate system below.



13. **a.** What is the slope of line *m*? What is its *y*-intercept?

b. Write an equation for line *m*.

c. Measure angle α (the Greek letter alpha).

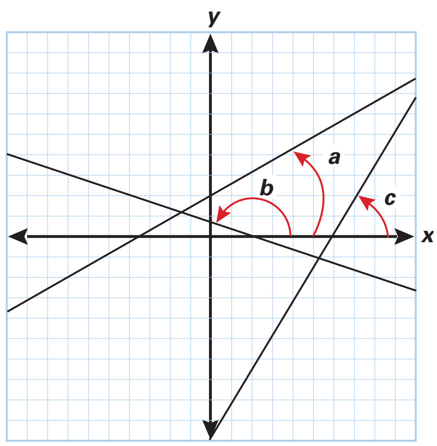
14. **a.** On graph paper, draw two lines in a coordinate system like the one here—one that forms a 30° angle with the *x*-axis and one that forms a 60° angle with the *x*-axis.

b. Estimate the slope of each line.

If both axes are scaled in the same way, there is an angle that corresponds to every slope. The slope is then equal to the **tangent ratio** for that angle, abbreviated *tan*.

$$\text{slope} = \tan \alpha = \frac{\text{vertical component}}{\text{horizontal component}}$$

15. Find the slope and measure the angle for each of the lines in the grid below. Note: The axes are scaled in the same way.



Reaching All Learners

Intervention

Some students find it easier to work with the equations that are not in simplest form. For example, they may prefer to write $y = 0 + 1x$ instead of $y = x$. Although the equation is not simplified, some students can directly see the values for the slope and y-intercept, so this form makes sense to them. Other students may simplify their equations, but do not force this at this time.

Vocabulary Building

Have students add the term *tangent ratio* to the vocabulary section of their notebooks.

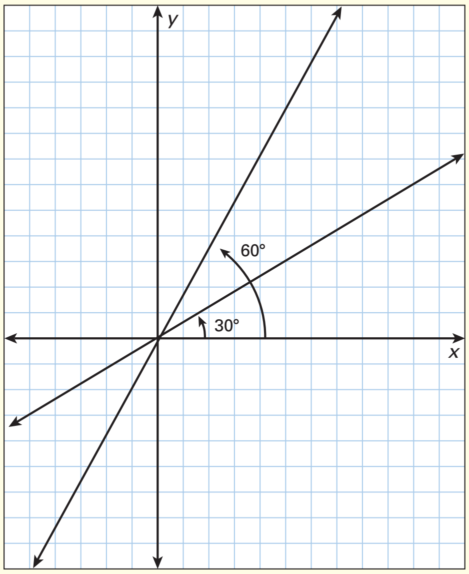
Solutions and Samples

13. a. The slope is 1, and the y-intercept is -2.

b. $y = -2 + x$

c. 45°

14. a. Lines may vary, depending on their y-intercepts. Sample graph:



b. Estimates will vary. Sample estimates:

Slope of the 30° line: $\frac{6}{10}$

Slope of 60° line: $\frac{7}{4}$

15. tan a = slope = $\frac{4}{7}$; angle a is about 30°.

tan b = slope = $-\frac{1}{3}$; angle b is about 160°.

tan c = slope = $\frac{5}{3}$; angle c is about 60°.

Hints and Comments

Materials

graph paper (one sheet per student);
protractors or compass cards (one per student);
ruler (one per student)

Overview

Students draw lines with given slopes and write equations for lines.

About the Mathematics

The tangent of the angle a line makes with the positive x-axis is the same as the slope of the line only if both the axes are scaled in the same way. The slope is not proportional to the angle; an angle that is twice as large does not correspond to a slope that is twice as large. The convention in mathematics is always to measure the angle with the positive (or right) part of the x-axis, to avoid confusion about what angle to deal with. The mathematics on this page will be developed further in the grade 8 unit *Looking at an Angle.*

Comments About the Solutions

13.–15. Students should use a protractor or compass card to measure and draw the angles. They should only measure the size of the angle formed with the right side of the x-axis.

14. b. Students estimate the slope of each line rather than find the exact slopes. One strategy is to compare the lines to lines with a slope the students know. A sophisticated strategy is to find a point where the lines cross the grid and use it to estimate the slope. For example, the line at a 60° angle crosses the grid at about (4, 7). This gives a vertical component (measured from 0) of 7, and a horizontal component of 4. The estimate for the slope would therefore be: $\frac{7}{4} = 1.75$. Discuss with students if an angle twice as large corresponds with a slope that is twice as large.

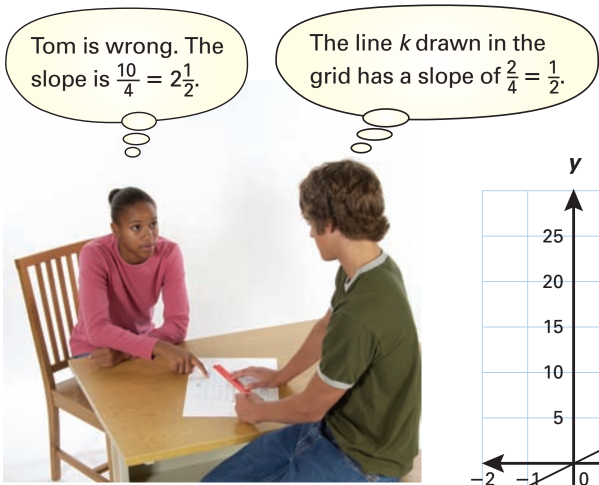
15. Students can find the slope of each line the same way as in problem 12 on page 23. They can use the rule above problem 15 to find this is the tangent ratio. The tangent can also be found by measuring the angle or using the (tan) key on a calculator to find the tangent ratio for a given angle.

An Equation of a Line

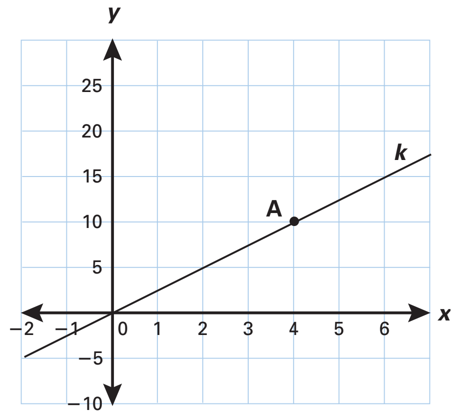
Notes

16 Students need to compute the vertical and horizontal change to find slope. Students often count squares on the grid to do this, but this works only when both axes have the same scale. Be sure to discuss this as a class.

19 Look for multiple strategies for discussion.



A grid does not always have the same scaling on both axes. If the scales are different, you cannot use the tan of the angle as slope.



16. **a.** Do you agree with Tom or Brenda? Explain your answer.

b. In your notebook, copy the grid above and draw a line through (0, 20) with slope −1.

c. Write the equation of the line you drew in part **b**.

17. **a.** Measure the angle that line k makes with the x-axis.

b. Draw a grid with equal scaling on both axes and draw a line k, through O (0, 0) and A (4, 10), in the grid.

c. Measure the angle that line k makes with the positive x-axis in the grid you drew for part **b**.

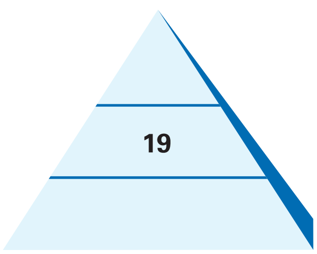
d. Which of the two angles you measured for line k, the one in part **a**, or the one in part **c**, corresponds to the slope? Give reasons for your answer.

18. **a.** What can you say about a line and its slope if the angle is 0°?

b. Will the angle that a line makes with the positive x-axis be greater than 90°? Explain your thinking.

19. If a line goes through (2, 3) and has slope 4, how could you find the y-intercept?

Assessment Pyramid



Understand the graph of a line in the coordinate plane.

Reaching All Learners

Advanced Learners

Ask advanced students to draw some angles larger than 90° on graph paper. They can try angles in all four quadrants. Be sure they measure all angles with 0° on the right-hand side of the x-axis. Students should find or approximate the slope from the graph and then find the tangent of the angle. Ask them to summarize the results.

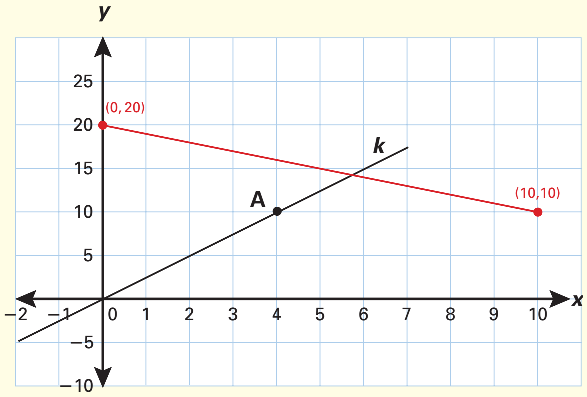
Extension

You may want to have students write the equations for the lines in some of the graphs from previous sections of this unit.

Solutions and Samples

16. a. Brenda is right. Explanations will vary. Sample explanation: To find the slope, that can be used in the equation, you have to use the correct units as indicated on the scaled axes as Brenda did. Tom used the grid size which is incorrect. He just "counted" 4 grid steps to the right and two up.

b.



c. Equation of this line is $y = 20 - x$.

17. a. The angle is about 27°.

b. Student graph should be similar to the graph shown on student page 25, with a different scale.

c. The angle is about 70°. (An exact answer would be 68°, but this measurement depends on the accuracy of student graphs.)

d. Sample answer: The answer from part c should be the angle measure that corresponds to the slope. In both graphs the change in y is +10, and the change in x is +4. But part c shows the correct slope angle since the scales for the x and y axis are the same.

18. a. If the angle a line makes with the positive x-axis is 0°, the line is parallel with the x-axis (or it is the x-axis itself).

b. Yes, the angle can be greater than 90°. Explanations will vary. Sample explanations:

- This is shown in the illustration for problem 15.
- To find the angle with the positive x-axis you start measuring from there and measure counterclockwise. So sometimes you have to go over a 90° if the line slopes from left to right downward.

Hints and Comments

Materials

graph paper (one sheet per student):
protractors or compass cards (one per student):
ruler (one per student)

Overview

Students relate the slope of a line to the angle the line makes with the x-axis. They investigate this relationship.

About the Mathematics

If the axes on a grid are not scaled in the same way, the tangent of the measured angle a line makes with the positive x-axis is not the same as the slope. The calculated ratio of the vertical component and the horizontal component is still the slope. To find the correct values of the components, the scales on the axes must be correctly used. It is not correct to just count the squares in the grid.

Planning

Students may work on problems 16–19 individually. Problem 19 is optional.

Comments About the Solutions

16. a. Make sure students understand the difference between finding the value of the horizontal and vertical component by using the scales on the axes and by counting squares in the grid.

17. b. You may want to discuss that the tangent of the angle is the value when you calculate the ratio for the vertical and horizontal steps based on the grid size.

18. Students may suggest measuring the angle clockwise from the positive x-axis so the angle will be below 90°. This can be done, but then the angle must get a minus sign and negative angles have not yet been discussed.

19. Answers will vary since different strategies are possible. Sample answers:

- The line can be graphed; from the graph the y-intercept can be found where the line crosses the y-axis.
- A slope of 4 means: for 1 horizontal step to the right you have to go 4 steps up (to stay on the line). To find the y-intercept, you have to walk from (2, 3) to the point where $x = 0$; this is two steps to the left. This means 2 x 4 steps down from 3, so you end at a height of $3 - 8 = -5$, which is the y-intercept.

C An Equation of a Line

Notes

Read the Summary as a class, or use a strategy that requires students to read the Summary. Be sure students know and understand the formula for the equation of a line.

Point out that the general equation of a line is often written as y = mx + b where b is the y-intercept and m is the slope.

C An Equation of a Line

Summary

All straight lines can be determined by a point and a direction. The direction is called *slope*. An equation for the line that contains the point (0, 5) and has slope 3 is $y = 5 + 3x$.

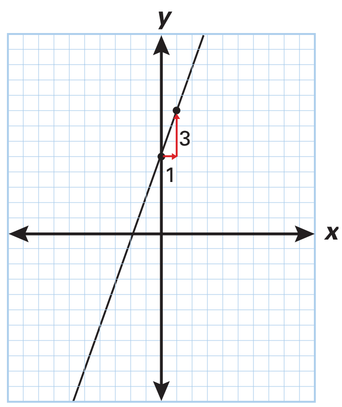
The number 5 indicates the **intercept** on the y-axis, which is a very special point.

The number 3 is the value of the slope.

The equation of a line that is not vertical has this form:

$y =$ intercept + slope • x

If you have the equation of a line, you can find the y-intercept by calculating the y-value for $x = 0$. The y-intercept of a line may be positive, zero, or negative. The slope of a line may be positive, zero, or negative.



Another way to describe the slope is using the tangent of the angle the line makes with the x-axis. You have to be careful with slope and tangents when the two axes in a grid are not scaled in the same way.

Reaching All Learners

Parent Involvement

Have students discuss the Summary problems with their parents, showing them examples in the section.

Hints and Comments

Overview

Students read the Summary, which reviews the main concepts covered in this section. They reflect on the equation of a line, the meaning of the y-intercept, and the slope and the relation between slope and tangent of the angle the line makes with the positive x-axis.

About the Mathematics

The notation used in the Summary for the equation of a line is referred to as the *slope intercept form*.
In mathematics, the general equation of a line is often written as $y = mx + b$. Vertical lines have no slope, and their equations are of the form $x = a$, where a is some constant.

C An Equation of a Line

Notes

1b Not all students will write the equation in simplest form.

3 A few students might find it easier to draw the line first and then write the equation.

4 Many students find it easier to use graph paper than to draw a coordinate system in their notebook.

Check Your Work

1. **a.** What is the y-intercept of the line with equation $y = -3 + 2x$? What is its slope?

   b. Write the equation for a line that goes through (0, 0) and has the same slope as the equation in part **a**.

2. **a.** Write an equation for the line through C (0, 15) with slope $-\frac{1}{4}$.

   b. Write an equation for the line through A (0, −5) with slope −1.

3. Draw a coordinate system in your notebook. Draw the lines defined in problem 2 in this coordinate system.

4. **a.** Write an equation of a line with a positive y-intercept and a negative slope. Draw this line in a coordinate system.

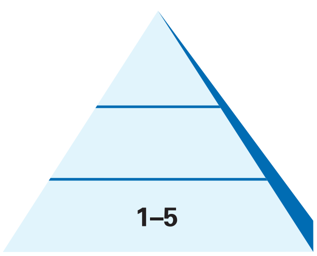
   b. What can you tell about a line with a y-intercept equal to 0?

   c. What can you tell about a line whose slope equals 0?

For Further Reflection

Describe in your own words what is meant by the word *slope*. In your description, also explain why it is important to be careful when finding the slope if the scaling on the x-axis is different from the scaling on the y-axis. You may use one or more examples in your description.

Assessment Pyramid



Assesses Section C Goals

Reaching All Learners

Parent Involvement

You may wish to have students show parents their work from this section and explain how they did the Check Your Work questions.

Solutions and Samples

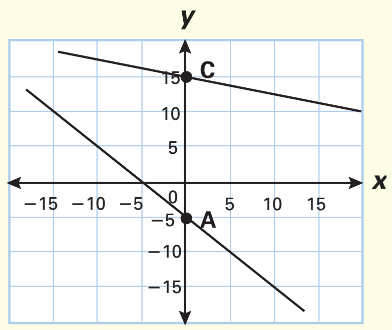
Answers to Check Your Work

1. **a.** The y-intercept is -3 and the slope is 2.

   b. If a line goes through (0, 0), the y-intercept is 0. So the equation is $y = 0 + 2x$ or even shorter $y = 2x$.

2. **a.** $y = 15 - \frac{1}{4}x$ or $y = 15 + (-\frac{1}{4})x$

   b. $y = -5 - 1x$
   or $y = -5 - x$

3. 

   You may have used different scales on the axes. If so, your line will look different.

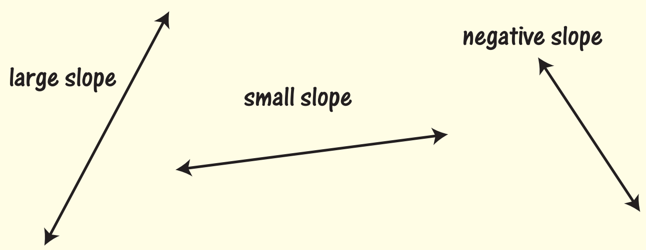
4 **a.** Many answers are possible; an example is $y = 2 - 3x$. When you draw a line with a positive y-intercept and a negative slope, it will always cross the y-axis above the origin, and it will run down to the right from there.

   b. A line with y-intercept 0 goes through O (0, 0).

   c. A line with slope 0 is a horizontal line since the vertical direction in the slope ratio must be 0.

For Further Reflection

Student answers will vary. Sample answer:
The slope is the amount of steepness in a line. Steep lines going uphill have a large slope. Lines that are almost flat have a very small slope. Lines going downhill have a negative slope.



You need to be careful if the scales on the x and y-axis are different. You need to make sure to use the points, like (1, 2) and (3, 6), to determine the slope. If the scales are different and you count along the grid lines, the slope will be incorrect.

Hints and Comments

Overview

Students work on the Check Your Work and For Further Reflection problems.

These problems are designed for student self-assessment. A student who can answer the questions correctly has understood enough of the concepts taught in the section to be able to start the next section. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work.

Answers are provided in the back of the Student Book. Have students discuss their answers with classmates.

Section Focus

Students investigate solving equations using diagrams, number lines, and symbols. They first do so informally in a context operation involving two imaginary frogs. The unknown that students have to find is the length of one single frog jump. The context helps set up a model involving diagrams and equations, which leads to a method for solving linear equations by canceling values on either side of the equation or otherwise stated: by performing the same operation on both sides of the equation.

Pacing and Planning

Day 11: Jumping to Conclusions		**Student pages 28–31**
INTRODUCTION	Problems 1–3	Investigate a context involving jumping frogs and compare the effect of different jump lengths on the distance that two frogs travel from starting points.
CLASSWORK	Problems 4–9	Use diagrams and equations to determine the unknown length of a frog jump.
HOMEWORK	Problems 10–12	Informally solve equations of the form $a + bx = c + dx$ using "frog problem" diagrams.

Day 12: Opposites Attract		**Student pages 31–33**
INTRODUCTION	Review homework.	Review homework from Day 11.
CLASSWORK	Problems 13–15	Solve "frog problems" that involve jumps in opposite directions.
HOMEWORK	Problems 16 and 17	Use diagrams to represent expressions and equations and solve an equation.

Day 13: Number Lines		**Student pages 34–37**
INTRODUCTION	Problems 18 and 19	Use a number line to represent and solve equations.
CLASSWORK	Problems 20 and 21	Operate with symbols to solve equations.
HOMEWORK	Check Your Work For Further Reflection	Student self-assessment: Solve equations using the "frog jumping" method.

Additional Resources: *Algebra Tools*; Additional Practice, Section D, page 47

Materials

Student Resources
No resources required.

Teachers Resources
No resources required.

Student Materials
Quantities listed are per student.

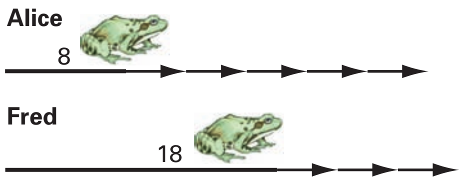
- Centimeter rulers

* See Hints and Comments for optional materials.

Learning Lines

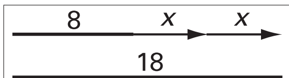
Solving Equations—Informally Using Diagrams and a Number Line

In the context of Alice and Fred, two frogs who are jumping away from a path, students consider what information they would need to determine Alice's and Fred's distances from the path after both have made a number of jumps of equal length from a given starting position. A diagram is used to clarify and structure the situation.

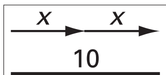


If both Alice and Fred end at the same distance and take jumps of equal length, the length of such a jump can be found. This can be done informally by reasoning from the situation. Students solved systems of equations informally in earlier units such as the grade 6 units *Comparing Quantities* and *Expressions and Formulas*.

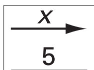
The length of the jump, the unknown, can be represented by a variable, in this case x. This leads to an equation for the problem: $8 + 5x = 18 + 3x$. The x can appear in both the diagram as well as the equation.



$8 + 2x = 18$



$2x = 10$



$x = \frac{10}{2} = 5$

Students first use the diagram, still related to the context, to solve the problem of finding the length of each jump (the value of x). These diagrams actually visualize equations. In the context it makes sense to cancel overlapping jumps and distances in the diagrams. Simultaneously with the changes in the diagrams during the solving, the equations change accordingly. This prepares students for the more formal way of solving equations by operating on the symbols.

Not only are equations with addition signs in them modeled with diagrams of the frog, subtraction can also be modeled if the frog jumps in the opposite direction. Another model is also used to represent and solve equations: jumps on the number line. This model allows for both positive and negative starting points (the numbers in the equations) as well as for positive and negative direction of jumps (the sign for the x-part in the equation).

Solving Equations—Formally

By simultaneously changing the diagrams and the equations the diagram visualizes to solve a problem, students learn to understand and use a formal way of solving equations.

$15 + 8x = 37 - 3x$
Add $3x$ to both sides.
$15 + 11x = 37$
Subtract 15 from both sides.
$11x = 22$
Divide both sides by 11.
$x = 2$

In an equation, the same number can be added or subtracted from both sides without changing the answer (the x-value). Also, the same number can be multiplied or divided from both sides without changing the answer.

Students learn to write down the operations they perform to keep track of the steps they take in solving the equations.

At the End of This Section: Learning Outcomes

Students can solve equations of the form $a + bx = c + dx$, where a, b, c, and d can be positive or negative.

Students can choose an appropriate way to solve equations: use a diagram, a number line or the formal method of operating on the symbols. Students know how to visualize an equation and reversely how to write an equation for a problem presented visually.

Notes

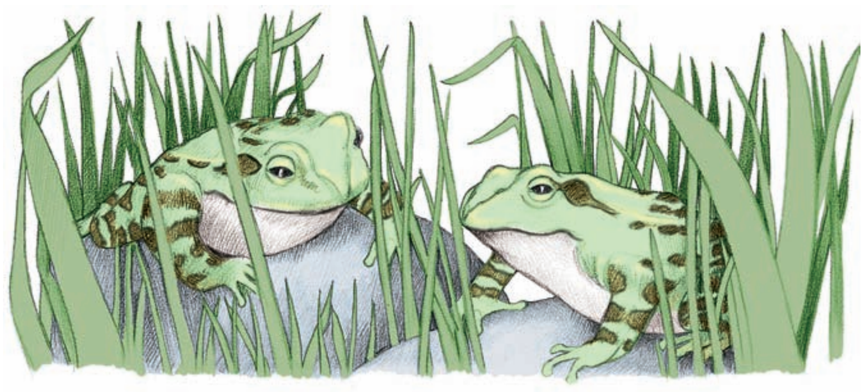
1 Be sure students understand that both frogs are jumping in the same direction, away from the path.

D Solving Equations

Jumping to Conclusions

The activities in the previous sections involved coordinates and directions. The activities led to investigating the slope and equation of a line. This section takes a look at writing and solving equations.

Two frogs, Alice and Fred, are near a path in a forest. Suddenly they hear footsteps on the path. To avoid possible danger, they jump away from the path.



Alice begins 8 decimeters (dm) from the path, and Fred begins 18 dm from the path. (Note: 10 decimeters = 1 meter.) Each frog takes several jumps and then stops.

1. What information would you need to find their new distances from the path?

Reaching All Learners

Act It Out

For a fun change of pace, use frog puppets to act out the first problem(s).

Solutions and Samples

1. You need to know how long a jump is, how many jumps each frog makes, and the directions of the jumps.

Hints and Comments

Materials

blank transparency (optional, one per class);

Overview

Students meet Alice and Fred, two frogs who are jumping away from a path. Students consider what information they would need to find the frogs' new distances from the path.

About the Mathematics

In the previous sections, students found the point of intersection of two lines graphically, reading the coordinates on the coordinate grid system. Finding the point of intersection of two straight lines, can also be done algebraically by solving a linear equation of the form: $ax + b = cx + d$.

In this section, informal and formal methods for solving this type of equation are developed. This process of solving equations is useful in many other algebraic problems. In the next section, the connection between solving these types of equations and (graphically) finding the point of intersection of two lines is made.

Planning

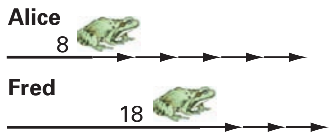
Students may work on problem 1 as a class.

Comments About the Solutions

1. You may want to introduce this section by telling the story about the jumping frogs and drawing the situation on an overhead projector or a blackboard. Draw a path as the starting point, and indicate the position where each frog starts jumping. Make clear that the frogs jump perpendicular to the path. You can use strips of equal length to indicate the jumps.

Notes

Suppose that Alice and Fred travel the same distance with each jump, but Alice takes 5 jumps and Fred takes 3 jumps. The diagram below illustrates their new positions.



Suppose Alice and Fred travel 4 dm with each jump.

2. a. Find the distance from the path to each frog's new position. Draw a diagram showing this situation.

b. Suppose you know that each jump is between 2 dm and 6 dm. What can you conclude about where each frog finishes?

Suppose the frogs finish their jumps at exactly the same distance from the path, and you want to know the distance of each jump and each frog's final distance from the path.

3. Write down your thinking about this problem. Share your group's method with the other members of your class.

3 Discuss student solutions. Look for different strategies.

Reaching All Learners

Intervention

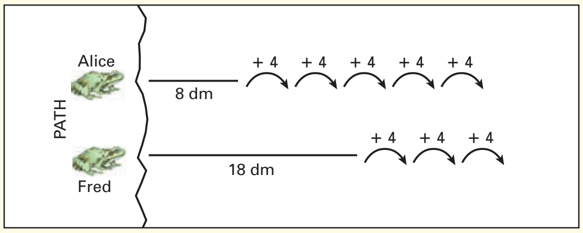
Be sure to continue to model both the diagram and the equation throughout this section. Do not encourage students to move to the formal level before they are ready. They will have plenty of opportunity to do this in future courses.

Advanced Learners

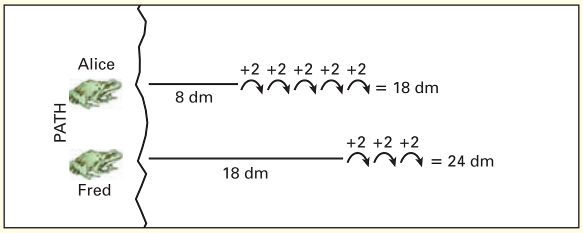
Some students will see the connection between the diagram and equation and formalize the relationship very easily. Allow students to move to more formal levels of representation, as they are able and interested.

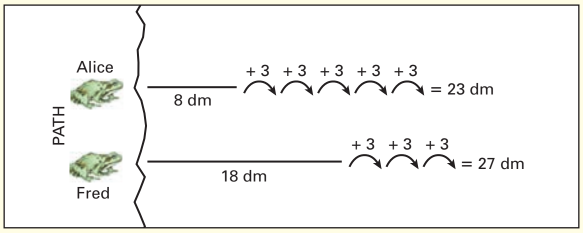
Solutions and Samples

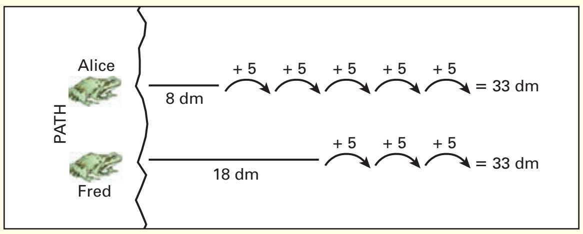
2. a. Alice will be 28 dm from the path, and Fred will be 30 dm from the path.

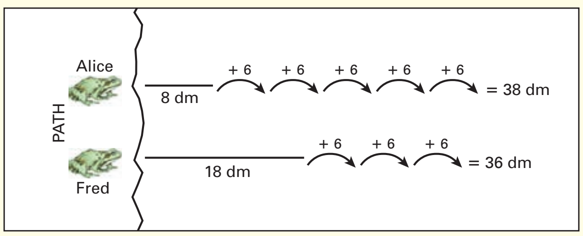


b. Answers will vary. Students may investigate only the resulting distance for the shortest and longest jump, or they may investigate all jump lengths. The following is a jump-by-jump summary of different possible jump lengths:









3. Answers will vary. Some students may say that 8 dm + n jumps should equal 18 dm + m jumps, where n is the number of jumps for Alice, and m is the number of jumps for Fred. The jumps should all be the same length. The following is a possible way to solve this problem.

Hints and Comments

Materials

centimeter rulers (one per student)

Overview

Students consider the effects of different jump lengths on the total distance each frog jumps. They explore diagrams and equations that describe the frog jumps and investigate how these can be changed according to the steps taken to find the length of each jump. This is continued on the next page.

About the Mathematics

The distance each frog jumps from the path depends on the starting position, the length of a jump, and the number of jumps. The distance jumped is called the *dependent variable*, because it depends on other variables. When the starting distance and the number of jumps are fixed, the only variable left is the length of a jump. It is common in mathematics to use a letter to represent a variable. Here the letter x is chosen. In the frog problems, the relationship between the dependent variable and the other variables can be written as follows: *total distance* = *starting distance* + *number of jumps* × *length*. This is a linear relationship.

In problems 2 and 3, the length and the number of jumps are variable. From right after problem 3, the length of the jump remains fixed for each situation but continues to be an unknown length. In the rest of this section, the number of jumps and the starting distance will be given. Because it is also given that the two frogs end at the same distance from the path, the situation is now completely fixed. The students' task is to find the value of the unknown length of a jump.

Comments About the Solutions

2. The starting distances are the same in both situations. Remind students to draw the diagrams to scale. For example, 1 cm represents 2 dm. If so, the starting distance for Alice would be 4 cm, and for Fred, 9 cm. The jumps should also be drawn to scale so students can measure their drawings to find the distance of each new position.

3. At this point, students may start to use shortcuts to describe situations. They may use letters to indicate the number of jumps and the length of jumps. Make sure students describe their general reasoning and not just the answer.

- Choose a value for the length of a jump and then find m and n for which the final distances are equal. Note that this is done in the solutions for problem 2b. From these students may conclude the jump length is 5 dm.

D Solving Equations

Notes

4 The model illustrated in boxes A–D is crucial. You may want to make an overhead transparency of these boxes to use during class discussion. Students may want to label each side of the equation with the name of the corresponding frog.

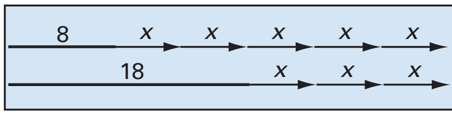
4, 5, and **6** Be sure students see how the diagram and the equation each change, and how they are still equivalent to each other. Encourage students to describe the steps taken from box to box in their own words.

7 These problems are connected to problem 2 on the previous page. Encourage students to describe the steps taken from box to box in their own words.

One way to answer problem 3 is to label the missing value, or the **unknown**. In this problem, the unknown is the length of each jump. You can use the symbol x for the length of a jump. The box below gives a diagram and an equation for answering problem 3.

Box A

4. Explain how the equation $8 + 5x = 18 + 3x$ describes the diagram in Box A.

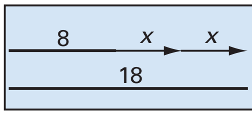


$8 + 5x = 18 + 3x$

Study the diagrams and equations to see the steps in finding the length of a jump.

Box B

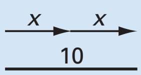
5. Explain the equation in Box B and describe how the diagram was changed from Box A to Box B.



$8 + 2x = 18$

Box C

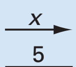
6. Explain the equation in Box C and describe how the diagram was changed from Box B to Box C.



$2x = 10$

Box D

7. Explain the equation in Box D and describe how the diagram was changed from Box C to Box D.



$x = \frac{10}{2} = 5$

Reaching All Learners

Intervention

Some students will solve equations using only symbolic manipulation. Be sure to use both diagrams and equations when you model solutions. Some students need to use the diagrams for the equations to have meaning. If they move to the formal level too quickly, they use procedures with little understanding and choose incorrect procedures more frequently.

Vocabulary Building

Have students add the term *unknown* to the vocabulary section of their notebooks.

Solutions and Samples

4. Explanations will vary. Some students may explain that the expression $8 + 5x$ represents the total distance Alice jumped. She started at 8 dm and took five jumps of length x. The expression $18 + 3x$ represents the total distance Fred jumped. He started at 18 dm and took three jumps of length x. If both Alice and Fred end up at the same distance from the path, the two expressions must be equal.
5. The equation shows that the starting distance of 8 dm plus 2 jumps of unknown length (x dm) equal 18 dm. Three jumps have been taken away from each frog.
6. Two jumps of length x equal a distance of 10 dm. Eight decimeters have been taken away from each frog's starting distance.
7. If two jumps equal 10 dm, then each jump equals $\frac{10}{2} = 5$ dm. Half is taken from each distance in box C to get box D.

Hints and Comments

Overview

Students explore diagrams and equations that describe the frog jumps and analyze one strategy for solving an equation. This is a continuation from problem 3. Students write frog problems, expressions and equations for diagrams and find the value of x by completing a sequence of diagrams and equations.

About the Mathematics

The diagram and the equation for the frog problem are changed step by step to find the unknown value represented by x (the length of a jump). This is a pre-formal introduction to solving equations by performing the same operation on both sides of the equation and canceling overlapping distances and jumps.

Students write frog problems and equations to fit diagrams. They change both the diagram and the equation step by step in order to find the value of x.

Planning

Students may work on problems 4–7 individually.

Comments About the Solutions

4. This problem is critical since it connects the equation to the diagram and the context. It is the first time that the representation of equations with arrows combined with x's is introduced. It is important that students understand this notation so that they can fall back on it whenever they are solving equations. The left side of the equation represents the total distance traveled by one frog, and the right side of the equation, the total distance traveled by the second frog.

D Solving Equations

Notes

8a and **b** Note that students are representing an expression rather than an equation here.

10 Encourage students who have difficulty to model the equation with diagrams.

11 and **12** Encourage students who have difficulty to model the equation with diagrams.

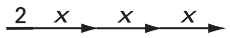
12 Some students will combine like terms. Others will draw diagrams. Students can rearrange the parts of their diagrams if this helps them.

12 Some students often attempt to use only symbolic language. Be sure they understand the reasoning behind their steps.

8. Write a "frog problem" and an expression to represent each diagram in parts **a** and **b**.

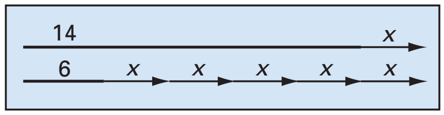
a.

b.



9. **a.** Write a "frog problem" and an equation to represent the diagram in Box A below.

Box A



b. Draw the diagram for the next step in finding the value of x and write the equation for your diagram.

c. Complete the sequence of diagrams and equations.

10. Here is an equation: $12 + 2x = 6 + 4x$.

a. Use a sequence of boxes to solve the equation. Start by drawing a diagram to represent each side of the equation.

b. Draw the rest of the boxes and diagrams to solve the equation.

11. **a.** Describe the equation $11 + 9x = 26 + 4x$ as a "frog problem."

b. Find the value of x in the equation and explain the steps in your solution. You may want to use a series of boxes, diagrams, and equations as part of your explanation.

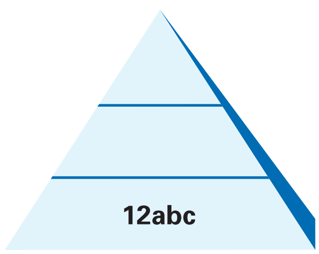
12. Solve each equation for its unknown value and explain your method. How can you be sure that your answers are correct?

a. $100 + w + w = 75 + w + w + w + w$

b. $y + 42 + y = 12 + 3y + 2y$

c. $144 + z = 120 + 9z$

Assessment Pyramid



Solve equations.

Reaching All Learners

Intervention

Many students benefit from a review of the steps used in solving equations. Be sure students see how the change from one step to the next is shown in the diagram and in the equation. Students need the connection between the diagram and equation to be explicit; be sure they can verbalize this. Emphasize the equality of distance as a way to represent the equality of expressions. Encourage the use of the diagram as a way to visualize the problem.

Solutions and Samples

8. a. Problems will vary. Sample problem:

A frog starts 10 dm from the path and takes four jumps of the same length. The diagram can be represented by the following expression: $10 + 4x$.

b. Problems will vary. Sample problem:

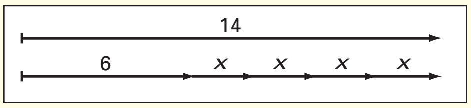
A frog starts 2 dm from the path and takes three jumps of the same length. The diagram can be represented by the following expression: $2 + 3x$.

9. a. Problems will vary. Sample problem:

Alice starts 14 dm from the path and takes one jump. Fred starts 6 dm from the path and takes five jumps of the same length as those of Alice. They end up at the same distance from the path.

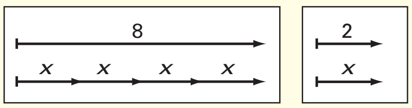
$6 + 5x = 14 + x$

b. $14 = 6 + 4x$



c. $8 = 4x$

$x = \frac{8}{4} = 2$ dm

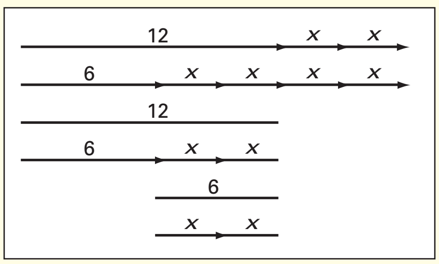


10. a.–b.

$12 = 6 + 2x$

$6 = 2x$

$x = 6 \div 2 = 3$



11. a. Descriptions will vary. Sample description:

One frog starts 11 dm from the path and takes nine equal jumps. Another frog starts 26 dm from the path and takes four equal jumps of the same length as those of the first frog. They both end up an equal distance from the path.

b. The value of x is 3 dm. Strategies will vary. Sample strategy, solving the equation for x:

$11 + 9x = 26 + 4x$

$11 + 5x = 26$

$5x = 15$

$x = 15 \div 5 = 3$

Hints and Comments

Overview

Students solve equations by drawing sequences of boxes and diagrams and by writing sequences of equations. They decide what method they want to use and explain their method they chose.

About the Mathematics

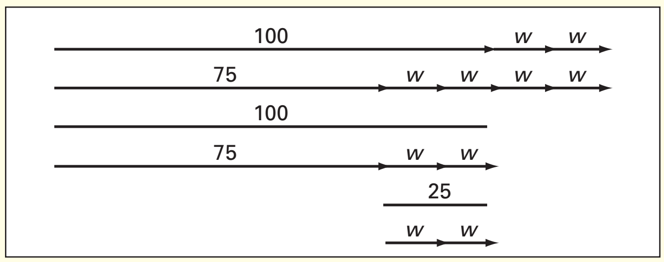
Students may use the box diagram to visualize the equation. The diagrams are drawn to scale, allowing for another solution strategy—measuring the length of x and multiplying it by the scale factor. It may be helpful for students who have "discovered" this measuring strategy themselves to use this strategy once so they will recognize the meaning of the visual model. However, these students should be encouraged to use the step-by-step method since this will lead to a method for solving these problems that also works for operating on the equations (as the measuring method will not).

Planning

Problems 10 and 11 may be assigned as homework, and problem 12 can be used as an informal assessment.

See more Hints and Comments on page 71.

12. a.–c. Strategies will vary. Sample strategies:



a. $100 + 2w = 75 + 4w$

$100 = 75 + 2w$

$25 = 2w$

$w = 12.5$

b. $42 + 2y = 12 + 5y$

$42 = 12 + 3y$

$30 = 3y$

$y = 10$

c. $144 + z = 120 + 9z$

$144 = 120 + 8z$

$24 = 8z$

$z = 24 \div 8 = 3$

D Solving Equations

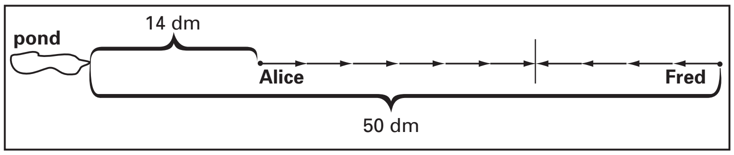
Notes

Problems 13 (on this page) and 14 (on the next page) are similar. They deal with the same situation. You may want to discuss these problems in class, to make sure all students can follow the steps from Box A to Box D. Encourage students to describe their solutions in their own words.

13b Be sure students visually see that Alice making 4 more jumps gives the same distance as Fred staying at 50 dm.

Opposites Attract

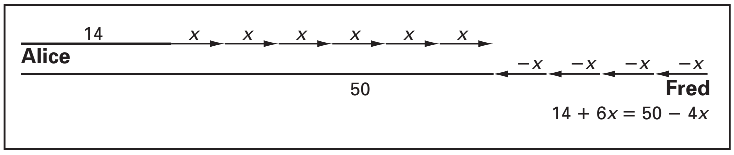
One day, while exploring their territory, Alice is 14 dm from the pond and Fred is 50 dm from the pond. They start jumping toward each other. As shown below, they met after Fred took four jumps toward Alice and Alice took six jumps toward Fred.



13. Suppose both frogs travel the same distance x with each jump.

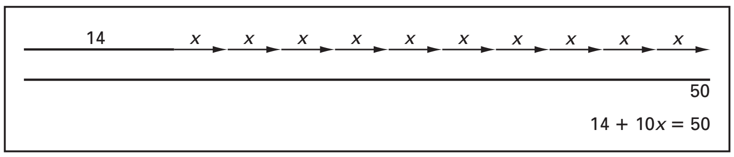
a. Explain how the diagram and equation in Box A represent the frogs' positions.

Box A



b. Explain the equation in Box B and describe how the diagram was changed from Box A to Box B.

Box B



Reaching All Learners

Accommodation

You may want to make an overhead transparency for problem 13 showing the sequence of boxes with the diagrams and equations to use during discussion.

Solutions and Samples

13. a. Explanations will vary. Sample explanation: The first diagram shows Alice starting 14 dm from the path and making 6 jumps. The second diagram shows Fred starting 50 dm from the path and making 4 jumps in the opposite direction. They both end up at the same place.

This is also shown in Box A. Note that the total distance in Fred's diagram is 50 (so including his jumps). The lengths of all the jumps of both frogs are equal, x, the minus signs in the diagram indicates that Fred jumps in the opposite direction from Alice. In the equation this is shown with a subtraction sign, Fred's jumps are subtracted from his starting distance.

b. Explanations will vary. Sample explanation: If Alice takes four more jumps, she will be at Fred's starting point (50 dm from the path).

The four jumps of Fred's in the negative direction were added to Alice's jumps in the positive direction.

So the starting distance of Alice (14 dm) plus 10 jumps of length x equal a distance of 50 dm.

Hints and Comments

Materials

transparencies of the three drawings on this page (optional)

Overview

Students solve equations and describe equations as frog problems where frogs now jump in opposite directions.

About the Mathematics

In all problems on this page, the starting distance is in the same direction for both frogs, but now both frogs jump in opposite directions toward each other. The length of the jumps is positive (of course); the minus sign indicates a jump is in the opposite direction. This model allows for equations with a negative term in x like: $14 + 6x = 50 - 4x$.

The diagram for this type of equation is a little bit more complex than the diagrams before because jumps now go back on the "line" representing the starting distance. Students were formally introduced to operating with positive and negative numbers in the unit *Operations*. In the context of jumps by frogs, the x always stands for the distance. Distance in this context has a direction (either positive or negative) and a positive length.

Planning

Students may work on problem 13 individually. This problem should be discussed with problem 14 on the next page.

Comments About the Solutions

13. a. Make sure it is clear to students that in Box A the starting distance of 50 cm of Fred is the total length, including Fred's "opposite" jumps.

D Solving Equations

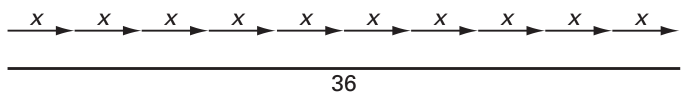
Notes

14 Discuss as a whole class. You may want to make an overhead transparency showing the sequence of boxes with the diagrams and equations to use during discussion.

17a Visual learners may want to make a diagram first. Accept visual, logical, and algebraic explanations.

14. a. Explain the equation in Box C and describe how the diagram was changed from Box B to Box C.

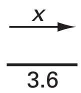
Box C



$10x = 36$

b. Explain the equation in Box D and describe how the diagram was changed from Box C to Box D.

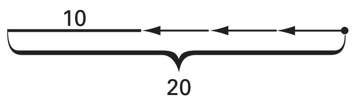
Box D



$x = \frac{36}{10} = 3.6$

15. Write a "frog problem" to represent Box A below. You can use Fred and Alice, or you may introduce new characters and situations. Be sure to solve your problem.

Box A

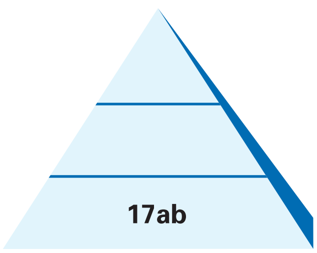


16. Draw a diagram to represent the expression $5 - 4x$.

17. a. If you start with the equation $27 - 5w = 7 + 3w$, explain why $27 = 7 + 8w$.

b. Solve the equation.

Assessment Pyramid



Explain equivalence and solve equations.

Reaching All Learners

Extension

You can have students make drawings similar to the one in problem 15. Let a classmate make up the story or the expression that goes with the drawing.

Intervention

For question 17b, encourage students who find solving equations challenging to use diagrams to model the steps. Model using diagrams when you demonstrate problems so students see this as a desired practice.

Solutions and Samples

14. a. Explanations will vary. Sample explanation:

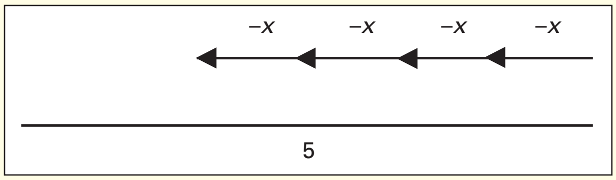
Ten jumps are equivalent to 36 dm. To get the diagram in Box C, 14 dm were subtracted from each length in Box B.

b. Ten jumps is 36 dm, so one jump is equal to $36 \div 10 = 3.6$ dm.

15. Problems will vary. Sample problem:

A frog starts 20 dm from the path and takes three jumps of the same length toward the path. It ends up 10 dm from the path.

16.



17. a. Explanations will vary. Some students may say that $5w$ has been added to both sides of the equation. Only the 27 remains on the left side, and on the right side $3w$ and $5w$ can be added to make $8w$. Other students may want to use a diagram in their explanation or use references to the frog problems. They may, for example, use terms like "starting distance"and "jumps in the opposite direction" in their explanation.

b. $27 - 5w = 7 + 3w$

$27 = 7 + 8w$

$20 = 8w$

$w = 20 \div 8 = 2.5$

Hints and Comments

Overview

Students solve and write frog problems in which frogs jump toward each other until they meet. This involves having a minus sign in the equation.

Planning

Students may work on problem 13 (on the previous page) and 14 in small groups. Problems 15–17 may be done individually.

Comments About the Solutions

14. This problem is connected to problem 13 on the previous page. These two problems are similar and deal with the same situation.

15. Students may also write an expression representing the drawing in Box A, although this is not explicitly asked for in the problem. You may want to discuss this in class. Note that rather than a "frog problem," students have to write a "frog story" for this diagram. Students can make their story into a problem by choosing a number for the final distance and asking for the length of the jump that results in this final distance.

17. After students have finished problem 17, you may want to discuss the different kinds of equations students have been solving so far and how to visualize them using box diagrams. Stress the importance of describing and understanding each step in the solution process.

If students have difficulty expressing what to do to cancel out the $-5w$ on the left side of the equation, you might remind them of the jumps. Tell students that five jumps to the left can be cancelled out by five jumps to the right. Use a drawing to illustrate this.

D Solving Equations

Notes

D Solving Equations

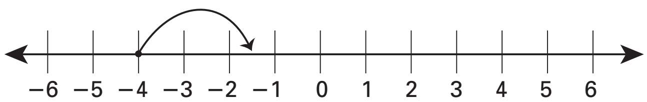
Number Lines

Frog problems can also be diagrammed on number lines.

A number line has a positive and negative direction.

Jumps on the number line are considered positive if they move in a positive direction and negative if they move in a negative direction.

Here is an example.



Starting point is −4; jump is +2.5

Starting point is −1; jump is −3

Fred starts at the point −3 and makes 17 jumps in a positive direction. Alice starts at the point 2 and makes 12 jumps in a positive direction. They end at the same point. Assume that every jump is the same length, and use the letter *k* for that unknown length.

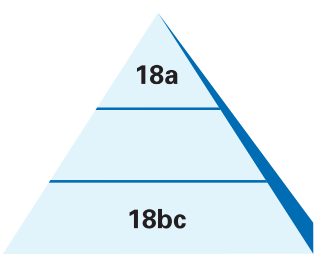
18. **a.** Write an equation for this situation.

b. Find the value of *k*.

c. Use a number line to check your solution.

18a Some students will find it helpful to make a diagram to visualize the situation before writing the equation.

Assessment Pyramid



Model a problem situation as an equation.

Reaching All Learners

Advanced Learners

You may want to challenge students who have an easier time solving equations to model a solution method using just the number line. Some might notice that the number of jumps becomes a divisor.

Solutions and Samples

18. a. $-3 + 17k = 2 + 12k$

b. $k = 1$. Sample strategy:

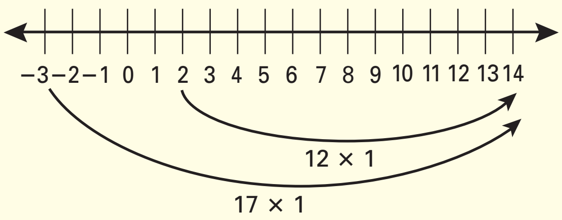
$-3 + 17k = 2 + 12k$

$-3 + 5k = 2$

$5k = 5$

$k = 1$

c.



Hints and Comments

Overview

Students draw diagrams to represent expressions and solve an equation. Then they are introduced to the number line as another model they can use to visualize the frog jumps.

About the Mathematics

Students now solve equations with minus signs in them. They may still use the drawings in the boxes; some students may be able to solve equations by operating on the equations itself. This more formal strategy gets attention later in this section. Number lines are used in many *Mathematics in Context* units, so students should be familiar with this model. Jumps to the left on the number line are indicated with a minus sign, while jumps to the right are positive. The number line has a dynamic character because it allows students to draw arrows that represent the moves that are made.

On this page the method for operating on the equations to solve them that was implicitly introduced by writing the equations in the boxes is now made explicit.

D Solving Equations

Notes

19a Students can use the "frog jump" model here also if they draw the jumps in the negative direction. This means that the x will be a negative number. You may want to discuss what the difference in meaning is between $-5x$ and $5(-x)$, but it is not necessary at this point.

Read through the text and the example as a class, or in groups. Ask students to explain the reasoning behind each of the steps used to solve the equation.

20 and **21** Students need to pay attention to operation signs as well as the coefficients. Also watch for integer computation errors.

19. The equation $8 + 12x = 3 + 2x$ represents a different "frog problem."

a. Use a number line to explain why the jumps must be in the negative direction.

b. Solve the equation.

In this section, you have solved equations using diagrams and a number line. Another method is to perform the same operation (addition, subtraction, multiplication, or division) on each side of an equation. Here is an example.

$15 + 8x = 37 - 3x$

Add $3x$ to both sides.

$15 + 11x = 37$

Subtract 15 from both sides.

$11x = 22$

Divide both sides by 11.

$x = 2$

20. Study the steps of the example shown above. Use similar steps to complete the solution for this equation.

$-5 - 6x = 1 - 9x$

Add $9x$ to both sides.

______ = ______

Add 5 to both sides.

______ = ______

____

______ = ______

21. Here are other steps to solve the same equation as in problem 20. Complete the solution and check to see if the result is equal to the result in problem 20.

$-5 - 6x = 1 - 9x$

Add $6x$ to both sides.

______ = ______

Subtract 1 from both sides.

______ = ______

____

______ = ______

Reaching All Learners

Intervention

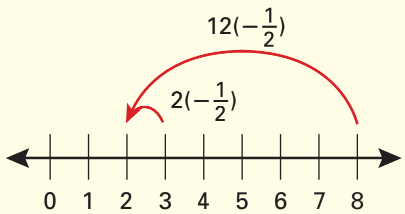
Solving the equation in problem 19 will not be easy for some students since a negative number will appear on one side of the equation. It may help to draw a frog diagram to see what steps in the equation make sense.

Students who have not developed a formal understanding of this process may need assistance with problems 20 and 21. When adding $9x$ to both sides in problem 20, for example, some students will add $9x$ to $6x$ to get $15x$. They focus on the coefficient of x rather than on the fact that $6x$ is being subtracted from $9x$. These students need help interpreting the meaning of the original equation to give meaning to the steps they are doing. Drawing a diagram to model the problem can also help.

Solutions and Samples

19. a. Answers will vary. Sample response:

The first frog starts at 8 and makes 12 jumps. The second frog starts at 3 and makes 2 jumps. If they both go in a positive direction, the first frog would begin with an advantage of 5 and make 10 more jumps. So it is impossible for the second frog to catch up with the first one.



b. $8 + 12x = 3 + 2x$

$8 + 10x = 3$

$10x = -5$

$x = -5 \div 10 = -0.5$

20. The completed solution is as follows:

$-5 - 6x = 1 - 9x$

Add $9x$ to both sides.

$-5 + 3x = 1$

Add 5 to both sides.

$3x = 6$

Divide both sides by 3.

$x = 2$

21. The completed solution is as follows:

$-5 - 6x = 1 - 9x$

Add $6x$ to both sides.

$-5 = 1 - 3x$

Subtract 1 from both sides.

$-6 = -3x$

Divide both sides by -3.

$2 = x$

The value of x in both cases is 2.

Extension

You can have two students model problem 19. Write the equation on the board. Designate a start position zero and ask students to imagine a number line. Student 1 starts at 8 on the imaginary number line. Student 2 starts at 3. Both face the same direction, toward the right. Connect the positions of the students back to the information in the equation. Ask if both take steps that are the same size, can person 2 take two steps and end at the same place where person 1 will be after 12 steps. It will be obvious that this cannot work. Have both students return to their start positions, but turn and face left on the number line.

Hints and Comments

Overview

Students use a more formal way to solve equations.

About the Mathematics

Equations of the form $a + bx = c + dx$, can be solved by performing the same operation on both sides of the equation. It helps keep track of the solving process to write the operations next to the equations. It makes clear what is actually done. This gives a better understanding of the process of solving equations than using "canceling out terms." This last way of speaking often confuses students since it is unclear what it means and what canceling out actually involves. Note that equations like this can be solved in numerous ways. For example, in problem 21 you can eliminate the x on the left side of the equation by adding $6x$, or eliminate the x on the right side by adding $9x$. The same method can be used for the starting distances: eliminate them from the right side by subtracting one, or from the left side by adding five.

Planning

You may want to draw the diagrams connected to the equations in the example to clarify what the steps mean in the context of the jumping frogs.

Comments About the Solutions

20. and **21.**
Make sure that students do not just follow the steps that are given, but also understand and can explain why they should follow these steps. These problems offer the opportunity to discuss why different methods can produce the same result (See About the Mathematics.).

21. Sometimes students get stuck in a circular process; for instance, they may add $3x$ to both sides as the next step and than subtract it again. If this happens, explain that the goal is to find the value for x, so have x at one side of the equation and a number on the other. You may encourage students to draw the diagram fitting the frog problem for the equation and see what operation makes sense if they want to find the length of the jump.

Student 2 moves from his start position of 3 two steps in the negative direction. Ask if person 1 (starting at 8) could take 12 steps to catch up with person 2. Students can see that this can work.

You may want to have students write the three problems, make answer keys for them, and exchange their problems with others in the class. Students can solve and check each other's problems.

D Solving Equations

Notes

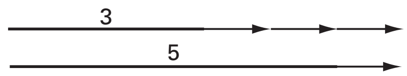
Read the Summary aloud as a class or in groups. Ask students to draw a diagram for the equation shown and each of the steps, or to explain the reasoning behind each of the steps. (Students can choose.)

D Solving Equations

Summary

In this section, you used frog problems to write and solve equations. You solved equations by drawing diagrams, by using number lines, and by performing an operation (adding, subtracting, multiplying, or dividing) on each side of the equation.

For example, frog A starts 3 dm from a log, and frog B starts 5 dm from the same log. Frog A and B take jumps that are the same length. Frog A takes 3 jumps and frog B takes 1 jump and they meet in the same location, as shown in the diagram below.



To find out how long the jumps were, you solve the equation for the problem.

$$\begin{aligned} 3 + 3x &= 5 + x \\ 3 + 2x &= 5 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

Reaching All Learners

Intervention

Encourage students to use diagrams to support their understanding of solving equations if they are having difficulty or are unsure what procedure to use. Do not push formal, symbolic methods before students are ready. They need to use procedures they understand.

Hints and Comments

Overview

Students read the Summary, which reviews the main concepts covered in this section.

About the Mathematics

It is important that students recognize the different methods for solving equations that they have learned in this section. Students may have a favorite method, but they should be able to use other methods as well.

Check Your Work

1. Write a “frog problem” and an expression to represent this diagram.

2. a. Draw a diagram representing this equation: $2 + 3x = 10 + x$.

   b. Solve the equation by changing the diagram step by step.

3. Here is an equation for a “frog problem” where the frogs are jumping in opposite directions and they meet.

$$20 + 2v = 26 - 2v$$

   Solve the equation. Then check your answer using a number line.

4. Solve each equation. You may use any method from this section.

   a. $12 + u = 11 + 3u$

   b. $-4 + 2w = 2 + w$

   c. $10 - v = 24 + v$

5. Write three new “frog problems”— one that you think is easy, one that is more difficult, and one that is very difficult. Describe how to solve each problem.

For Further Reflection

Think about the three different methods for solving an equation. What are the advantages and disadvantages of each method?

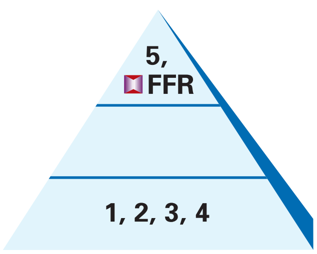
5 Students tend to think that using very large numbers makes problems more difficult. You can encourage them to write a problem in which both frogs jump in a positive direction, a problem in which the frogs jump in opposite directions, and a problem in which both frogs jump in the negative direction. It may help some students to draw diagrams for each situation first and then write the problems.

5 Students may enjoy exchanging problems and solving their colleague’s problems. You can solicit problems to use on a quiz.

For Further Reflection

Reflective questions are meant to summarize and discuss important concepts. Post student work from this problem.

Assessment Pyramid



Model a problem situation as an equation.

Solve equations.

Reaching All Learners

Advanced Learners

Advanced learners will probably use formal, symbolic procedures. Be sure they show their steps in solving equations.

Parent Involvement

Have students explain to parents or family members different ways to solve equations. Students can share their map and work they did on the For Further Reflection question. You may want the parent to write a short response to the student.

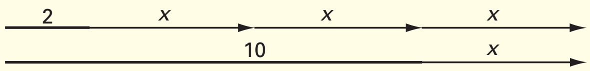
Solutions and Samples

Answers to Check Your Work

1. Answers may vary. Sample answer:

A frog starts 7 dm from the path and takes three jumps of the same length. The expression to represent the diagram is $7 + 3x$.

2. a.



b.

$$
\begin{aligned}
2 + 3x &= 10 + x \\
&\quad -x \\
2 + 2x &= 10 \\
&\quad -2 \\
2x &= 8 \\
&\quad \text{Divide by 2.} \\
x &= 4
\end{aligned}
$$

3. You can use different methods to solve the equation such as drawing frog diagrams, using a number line, or performing operations on both sides. Here is a sample solution using the method of performing operations:

$20 + 2v = 26 - 2v$

add $2v$ to both sides

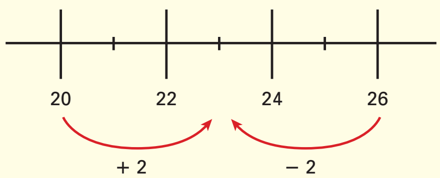
$20 + 4v = 26$

subtract 20 from both sides

$4v = 6$

divide both sides by 4

$v = 6/4 = 1.5$



4. a. $12 + u = 11 + 3u$
$12 = 11 + 2u$
$1 = 2u$
$u = 1 \div 2$
$0.5 = u$

b. $-4 + 2w = 2 + w$
$2w = 6 + w$
$w = 6$

c. $10 - v = 24 + v$
$10 = 24 + 2v$
$-14 = 2v$
$-14 \div 2 = v$
$-7 = v$

5. Answers will vary. This is a good opportunity for peer-assessment: have students trade papers and verify that the solution matches the problem.

Hints and Comments

Overview

Students work on the Check Your Work and the For Further Reflection problems. These problems are designed for student self-assessment. A student who can answer the questions correctly has understood enough of the concepts taught in the section to be able to start the next section. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work. Answers are provided in the Student Book. Have students discuss their answers with classmates.

Comments About the Solutions

4. This may be a good opportunity for students to share methods when they check their answers and discuss why different methods can still produce the same answer.

5. You may want to have students share their problems and solution methods.

For Further Reflection

Student responses should summarize the strategies for solving equations used in this section.

Sample response: To solve equations you could use frog jump pictures, using the same operation with equations, or a number line. Frog jumps are cool because you can see how it works, but they take a long time to draw. Equations are easy to write, but it is also easier to make mistakes with plus and minus signs. Using a number line is like frog jumping, you can see what is happening in each step. To use the number line you have solve the equation any way using the same operation method, but it helps for checking your answer.

Section Focus

Students make the connection between the graphic and algebraic methods of solving equations (or finding the point of intersection of two lines). Students found the point of intersection of two lines in a graph in Sections A, B, and C in the context of forest fires. In Section D, students learned a formal algebraic method for solving linear equations by performing the same operation on both sides of the equation.

Pacing and Planning

Day 15: Meeting on Line		**Student pages 38 – 40**
INTRODUCTION	Problems 1 and 2	Estimate the coordinates where two lines intersect and use the equations for the lines to check the estimate.
CLASSWORK	Problems 3–5	Solve the equations to determine the coordinates of the point where two lines intersect.
HOMEWORK	Problems 6 and 7	Find the line(s) that intersect through a point, using slope and y-intercept.

Day 16: Meeting on Line (Continued)		**Student page 40 – 41**
REVIEW	Review homework.	Review Day 15 homework.
CLASSWORK	Problems 8–11	Solve problems involving equations and intersecting lines.
HOMEWORK	Check Your Work For Further Reflection	Student self-assessment: Section E Goals. Find the point of intersection for pairs of lines.

Day 17: Summary		
REVIEW	Review homework.	Review Day 16 homework.
UNIT SUMMARY	Unit review	Review section summaries and key concepts emphasized in *Graphing Equations*.

Day 18: Unit Assessment		
UNIT ASSESSMENT	Unit Test	Assessment of Unit Goals.

Additional Resources: *Algebra Tools*; Additional Practice, Section E, page 47

Materials

Student Resources
Quantities listed are per student.

Student Activity Sheet 6

Teachers Resources
No resources required.

Student Materials
Quantities listed are per student.

- Graph paper

* See Hints and Comments for optional materials.

Learning Lines

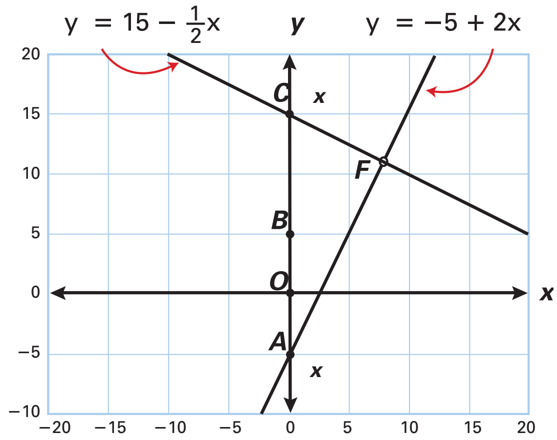
Point of Intersection and Solving Equations

In Sections A and B, students found the point of intersection of two lines by drawing them or reading the coordinates from the graph. In Section C, students have to find a point of intersection from a graph, where the grid is too small to show this point. Students need to reason using, for example, the direction or slope of each line. This is a numerical reasoning process.

In this section, students combine the graphic method to find a point of intersection with the use of equations. In the first problem in this section, they estimate the coordinates of the point of intersection graphically. Since the graph is not easy to read in detail, there is a need to check whether the point of intersection they found is indeed correct. Students therefore check the coordinates by using the equations. The equations here are just used for a check; students can fill in the coordinates for x and y and see if the equations result in true statements.

A graph of an equation like $y = 3x + 5$ is a representation of all the (x, y) points that are solutions to the equation. Therefore, when two lines are graphed, their point of intersection is the only point that solves both equations. By linking the lines in the graph to their equations using arrows, the method for solving frog problems by algebraically solving the equation from Section D, is related to finding the point of intersection of two lines. When solving the equation, students need to find the x-coordinate of the point of intersection, as well as the y-coordinate.

An example of this process is shown below.



The point of intersection of the two lines shown on the right is (8, 11). This can be read from the graph or determined by combining the equations: $15 - \frac{1}{2}x = -5 + 2x$.

Then solve for x, which is 8. Then plug the x value into one of the equations for y: $y = 15 - \frac{1}{2}(8) = 11$.

In the rest of the section, students use any method they like for finding points of intersection and solving equations. They connect the graphic and algebraic method explicitly for a deeper understanding of the two. Students also explore the relationship between parallel lines and graphs of lines without intersection points in this section.

At the End of This Section: Learning Outcomes

Students will be able to find and use equations for lines in the slope and y-intercept form. They can graph this type of equations in a coordinate plane as lines and are able to find the intersection point of two lines algebraically as well as graphically. Students understand the similarities between these two strategies.

They can choose an appropriate way to solve equations.

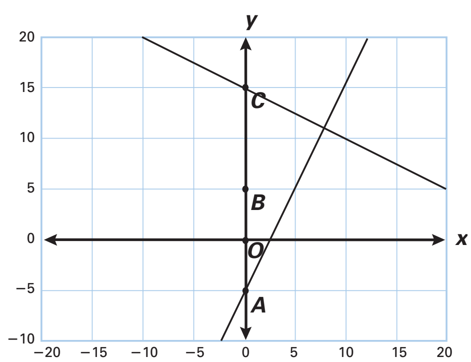
E Intersecting Lines

Meeting on Line

Let's return to the park rangers.

Rangers at tower *A* report a fire on the line whose equation is $y = -5 + 2x$.

Rangers at *C* report a fire on the line $y = 15 - \frac{1}{2}x$.



The two lines are displayed on a computer screen, as shown.

1. **a.** Explain how you can verify that the two lines on the screen represent the two equations.

   b. Using the screen, estimate the coordinates for the fire.

   c. How can you check your coordinates using both equations?

1c Not all students think of checking their solutions by substitution. Some may need prompting.

Reaching All Learners

Extension: Using Technology

Have students do the following graphing calculator activity after problem 1.

Set the window to the correct scale using the following values:

x min = −20; y min =−10; x max = 20; y max = 20; x scl = 5; y scl = 5

1. d. Using a graphing calculator, show the two lines from towers *A* and *C*.

e. Use the TRACE function of the calculator to move along one of the two lines. Then jump to the other line. What do you notice about the coordinates of the point as you jump from one line to the other?

f. What are the coordinates of the fire?

Students can also use the *intersect* or *table* features to solve the problem.

Solutions and Samples

1. a. Explanations will vary. One way is to check points on the line. Another is to check the slope and the y-intercept.

b. Estimates will vary, but the x-coordinate should be about 8, while the y-coordinate should be about 11.

c. The number chosen for x can be substituted into each equation. Both equations should give the same y value. For example:

If $x = 8$, then $y = -5 + (2 \times 8) = 11$,

and for the other equation:

$$y = 15 - (0.5 \times 8) = 11.$$

Therefore, 8 is the x-coordinate of the point of intersection for both these equations. The point of intersection is (8, 11).

If $x = 7$, then $y = -5 + (2 \times 7) = 9$,

and for the other equation:

$$y = 15 - (0.5 \times 7) = 11.5.$$

Since the x-coordinate 7 gives the y-coordinate 9 in one equation and the y-coordinate 11.5 in the other, it can't give the point that fits both

Hints and Comments

Materials

graph paper (optional, one sheet per student)

Overview

Students return to the context of park rangers who are locating fires. Students read from the graph and solve equations to find the intersection point of lines.

About the Mathematics

In this section, the relationship between the different representations for solving systems of linear equations is made explicit. Students find the point of intersection of two lines from a graph, solve a linear equation of the form $a + bx = c + dx$, and determine whether a value for x gives the same value for y in two different equations.

Comments About the Solutions

1. Throughout the section, be sure to stress the importance of making a sketch of the graphs and being able to explain the steps in the process of solving equations. Problem 1c is critical because students need to understand that the intersection point of two lines is the point that makes both equations true. It may be helpful to encourage students to discuss the similarities and differences between the fire problems and the frog problems.

E Intersecting Lines

Notes

2a The graph provides a visual reference for the steps in solving a system of equations.

3a Be sure students understand why the two parts of the equation are equal. If two things are equal to the same thing, they are equal to each other (the transitive property).

3b Students may use formal or informal strategies to solve the problem.

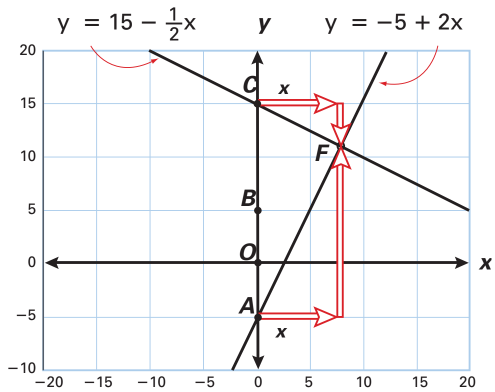
3c Some students have trouble relating their solution to part b back to the original equations.

What's the Point?

Here is another way to find the coordinates of point F, the intersection of the two lines.

Think about the change from point A to point F as a horizontal step followed by a vertical step. Suppose the length of the horizontal step is represented by x.

2. **a.** Write an expression for the length of the vertical step.



The change from point C to point F is the same horizontal step x followed by a vertical step.

b. Write an expression for the length of that vertical step.

From the diagram above, you can set up the following equation:

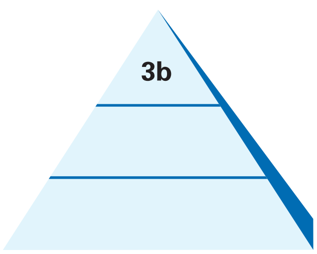
$$-5 + 2x = 15 - \tfrac{1}{2}x$$

3. **a.** Write a "frog problem" to go with the equation.

b. Solve the equation using one of the methods from the previous section.

c. How can you use your answer from part **b** to find the y-coordinate of F?

Assessment Pyramid



Choose an appropriate method to solve equations.

Reaching All Learners

Intervention

Some students will continue to use informal methods to solve equations. Diagrams can give meaning to the steps in solving equations for these students. You may need to help students see that the original equations provide instructions for finding the value of y if they know the value of x. Visual learners find it helpful to refer to the graphs of the lines and their point of intersection.

Vocabulary Building

Have students add the term *intersection* to the vocabulary section of their notebooks.

Solutions and Samples

2. a. The vertical step from point A to point F is $2x$ since the slope is equal to 2.

b. The vertical step from point C to point F is $-\frac{1}{2}x$ since the slope is equal to $-\frac{1}{2}$.

3. a. Answers will vary. Sample response:

Alice starts at -5 and takes two jumps of length x. Fred starts at 15 and takes $\frac{1}{2}$ jump of length x in the other direction.

b. $x = 8$. Students may use a variety of strategies to solve this problem. Sample strategy:

$-5 + 2x = 15 - \frac{1}{2}x$

$-5 + 2\frac{1}{2}x = 15$

$2\frac{1}{2}x = 20$

$5x = 40$

$x = 40 \div 5 = 8$

c. Substitute $x = 8$ into either of the equations. $y = -5 + 2 \times 8 = 11$. Point F is at (8, 11).

Hints and Comments

Materials

graph paper (optional, one sheet per student)

Overview

Students solve problems regarding the equation of a line, and solve equations to find the coordinates of the point of intersection of two lines.

About the Mathematics

Moves along a line can be described in steps or moves in both the horizontal and the vertical directions. In problems 2 and 3, the graphical way to find the point of intersection is connected to solving equations the algebraic way. The method for solving equations with the frog jumps is revisited to make the connection. One of the most important differences between solving frog problems and finding the point of intersection by solving equations is the fact that for the point of intersection the y-coordinate must be found as well. The x-coordinate of the point of intersection can be found directly by solving the equation; to find the y-coordinate students must fill in the x-coordinate in the equation for one of the lines.

Planning

Students may work on problems 2 and 3 individually.

Comments About the Solutions

2. If students have difficulty with this problem, you may want to revisit the use of direction pairs and the meaning of the slope of a line

3. In the Summary of Section D, students can look up the methods for solving equations.

3. c. This problem is critical since it makes clear how students can find the y-coordinate of the point of intersection. Discuss students' answers in class.

E Intersecting Lines

Notes

4a The tower location represents the y-intercept.

4b Some students will find it helpful to use the "frog jump" diagrams to help them solve the equations. Be sure students find both the x- and y-coordinates of the point of intersection.

5 This problem provides practice with integer and fraction computation. Watch for difficulties with computation when students are solving this system.

7 Some students may want to draw the lines first and then write the equations.

9 Students can only approximate the solution from the graph but can get an exact answer solving the equation algebraically.

The park supervisor has just received two messages:

Smoke is reported on the line $y = 15 - x$.

Smoke is reported on the line $y = 5 + 4x$.

4. a. Which tower sent each message?

   b. Calculate the coordinates of the smoke.

5. Repeat problem 4 for these two messages:

   Smoke is reported on the line $y = 5 + x$.

   Smoke is reported on the line $y = -5 + 1\frac{1}{4}x$.

The park supervisor received the message $y = 15 + 2x$ from tower C and the message $y = 5 + 3x$ from tower B.

6. What message do you expect from tower A?

7. Make up your own set of messages and find the location they describe.

Use **Student Activity Sheet 6** for problems 8 and 9.

8. a. Draw the line $y = 5$; label it l. Draw the line $y = -3 + 2x$ and label it m in the coordinate system.

   b. Find the point of intersection of the two lines on the graph; write down the coordinates.

   c. Use the equations of the lines to check whether the coordinates you found in **b** are correct.

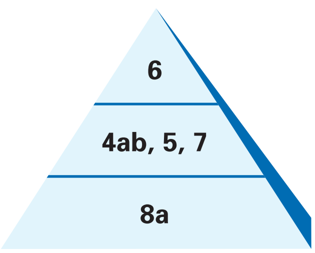
9. a. In the same coordinate system you used for problem 8, draw the line $y = 4 - 2x$; label it n.

   b. Estimate the coordinates of the point of intersection of lines m and n.

   c. Solve the equation $-3 + 2x = 4 - 2x$.

   d. Are your answers for **b** and **c** the same? Explain why or why not.

Assessment Pyramid



Model a problem situation.

Solve a system of equations algebraically.

Reaching All Learners

Intervention

For problem 6, some students will benefit from breaking the problem into parts and working each part sequentially. Many students will find it easier to find the slope of the line from tower A if they draw the line.

Many students benefit from connecting the symbolic, algebraic solution of a system of equations to the graphical representation of the point of intersection.

Solutions and Samples

4. a. Tower C reported smoke on the line

$y = 15 - x$ since tower C is at (0,15)

Tower B reported smoke on the line $y = 5 + 4x$ since tower B is at (0,5).

b. The smoke is at (2, 13). Strategies may vary. Sample strategy:

$15 - x = 5 + 4x$
$15 = 5 + 5x$
$10 = 5x$
$x = 10 \div 5 = 2$
$y = 15 - 2 = 13$

5. Tower B reported smoke on the line $y = 5 + x$. Tower A reported smoke on the line

$y = -5 + 1\frac{1}{4}x$

$5 + x = -5 + 1\frac{1}{4}x$

$5 = -5 + \frac{1}{4}x$

$10 = \frac{1}{4}x$

$40 = x$

$y = 5 + 40 = 45$

The coordinates are (40, 45).

6. The message from tower A should be:

$y = -5 + 4x$.

Students may use a variety of strategies to solve this problem. Sample strategy:

To find the point where the line from tower B intersects the line from tower C, graph the lines or make their two equations equal to each other as follows:

$15 + 2x = 5 + 3x$

$15 = 5 + x$

$10 = x$

$x = 10$

$y = 15 + 2 \times 10 = 35$

The point of intersection is at (10, 35).

The line from tower A at (0, −5) to (10, 35) has the slope.

$$\frac{35 - (-5)}{10} = \frac{40}{10} = 4$$

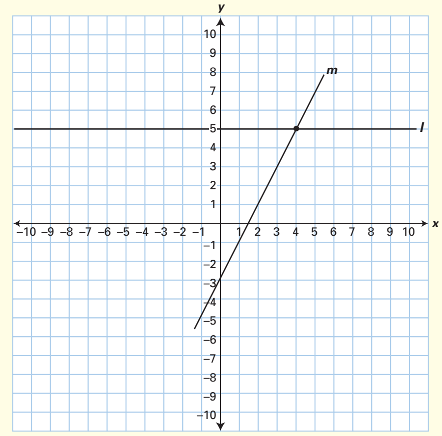
So the line from tower A must be $y = -5 + 4x$.

7. Answers will vary. Have students check each other's messages and location to see if they agree.

8. a. See the graph below.

b. Point of intersection is (4, 5).

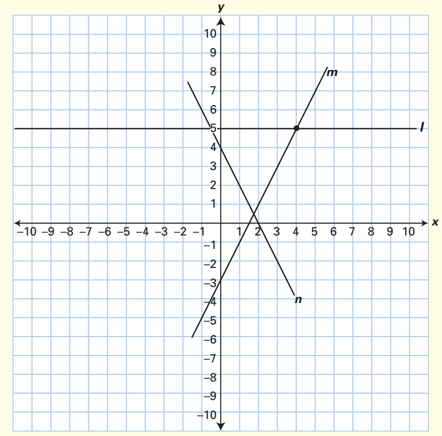
c. Sample answer:



Fill in the values $x = 4$ and $y = 5$ in the equations of the lines. For l, $5 = 5$, which is true, and for m you get:

$5 = -3 + 2 \times 4 = -3 + 8 = 5$, which is also true.

9. a.



b. Estimation of point of intersection: (1.7, 0.4)

c. Students may solve this equation using any method from the previous section.

Sample solution:

$-3 + 2x = 4 - 2x$

Add $2x$ to both sides

$-3 + 4x = 4$

Add 3 to both sides

$4x = 7$

Divide both sides by 4

$x = \frac{7}{4} = 1.75$

So $x = 1.75$ and $y = 4 - 2 \times 1.75 = 4 - 3.5 = 0.5$.
So the point of intersection is (1.75, 0.5).

d. The answers for **b** and **c** are close but not the same. This is because it is not possible to read off the coordinates exactly in such detail in the drawing of the graphs.

See Hints and Comments on page 72.

E Intersecting Lines

Notes

10 You may want to encourage students to draw the lines that are described by the equations. The graph clearly shows the lines are parallel and have no point of intersection. Connect what happens when students try to solve the equation algebraically to the graph.

Suppose the two lines $y = 10 + 2x$ and $y = -8 + 2x$ are on the park rangers' computer screen.

10. What can you tell about these lines? Do they have a point of intersection?

11. Look back at the graph for problem 20 on page 17.

a. Write an equation for each line in the graph.

b. Use the equations to find the coordinates of the point of intersection.

c. Compare the answer you found for part **b** to your answer from Section B.

Math History

Marjorie Lee Browne

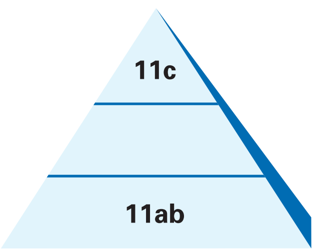
Marjorie Lee Browne loved mathematics and studied the subject to the highest possible standards. She was one of the first African-American women in the United States to obtain a Ph.D.

She was born on September 9, 1914, in Memphis, Tennessee. Her father, a railway postal clerk, was very good at mental arithmetic, and he passed on his love of mathematics to his daughter. Her stepmother was a schoolteacher.

Browne taught at Wiley College, in Marshall, Texas, from 1942 to 1945. She received her Ph.D. from the University of Michigan in 1949. She taught mathematics at North Carolina Central University. For 25 years she was the only person in the department with a Ph.D.

Browne used her own money to help gifted mathematics students continue their education. She will be remembered for helping students prepare for and complete their Ph.D.'s, encouraging them to do what she had achieved.

Assessment Pyramid



Understand similarities between graphic and algebraic strategies.

Solve equations.

Reaching All Learners

Extension

You may want to provide students with more equations and ask them to draw the corresponding lines and find the points of intersection either graphically or algebraically. If the coordinates of the point of intersection are not easy to read from the graph, it motivates the need for a symbolic algebraic method.

Solutions and Samples

10. Both lines have a slope of 2, so they are parallel and will not intersect.

11. a. The equation of line l is $y = 3 + \frac{3}{2}x$.

The equation of line m is $y = 2x$.

b. The point of intersection is (6, 12). Strategies will vary. Sample strategy:

$$3 + \frac{3}{2}x = 2x$$
$$3 = \frac{1}{2}x$$
$$6 = x$$
$$x = 6$$
$$y = 2 \times 6 = 12$$

The point of intersection is at (6, 12).

c. Answers will vary depending on students' previous answers. The answers should be the same.

Hints and Comments

Materials

graph paper (one sheet per student)

Overview

Students solve a variety of problems about intersecting lines.

About the Mathematics

In this unit, students use two major strategies for finding the point of intersection of two lines.
The first makes use of the graphs, and the second is the algebraic way for solving equations of the form $a + bx = c + dx$ dealt with in Section D. Students should be able to use both methods.

Extension: Using Technology

You may have students solve problems 9–11 using the graphing calculator. They can graph the equations and use intersect or table to find the point of intersection.

E Intersecting Lines

Notes

1 Some students may graph the lines. Some students may recognize they are given the coordinates of the point!

2c Students can verify the answer by solving the system of equations using formal or informal algebra strategies.

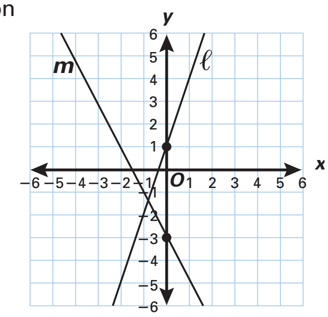
3 Students need to use fraction computation to solve the equation.

E Intersecting Lines

Summary

An equation for line ℓ is $y = 1 + 3x$, and the equation for line m is $y = -3 - 2x$.

You can try to find the point of intersection of lines l and m by reading the graph. Often this method will give you an estimate and not an accurate answer. Always use equations to check your result from reading the graph.



You can find the point where these two lines intersect by solving the following equation for x:
$-3 - 2x = 1 + 3x$

This will always give the exact result. You may use any method you used in Section D for solving the equation.

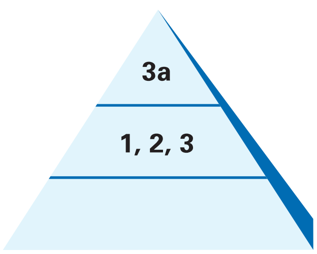
Check Your Work

1. What is the point of intersection of line $y = 3$ and line $x = -1$? How did you solve this problem?

2. a. Draw a coordinate system like the one on **Student Activity Sheet 6** and draw the line $y = -2 + 4x$.
   b. Write an equation for a line that has no point of intersection with the line from part **a**.
   c. Draw line $y = 4 + x$ in the same coordinate system and find the point of intersection of the two lines you drew. Explain how you know your answer is correct.

3. a. Find the x-value of the intersection of the lines shown in the Summary by solving the equation $-3 - 2x = 1 + 3x$.
   b. Find the y-value of the point of intersection of lines m and ℓ shown on this graph.

Assessment Pyramid



Assesses Section E Goals

Reaching All Learners

Study Skills

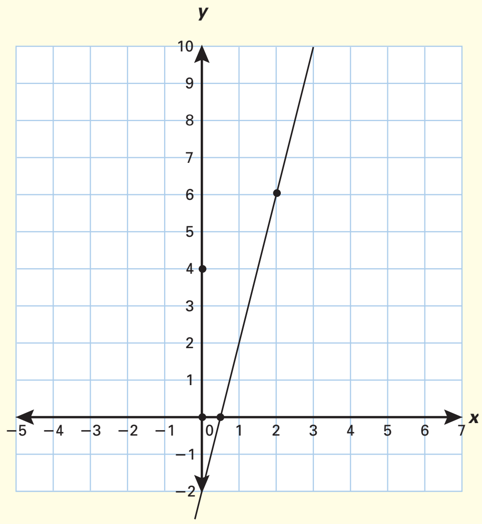
Ask students to outline the steps used to find the point of intersection using algebra. Have students read the Summary. Did they leave out any important steps?

Solutions and Samples

Answers to Check Your Work

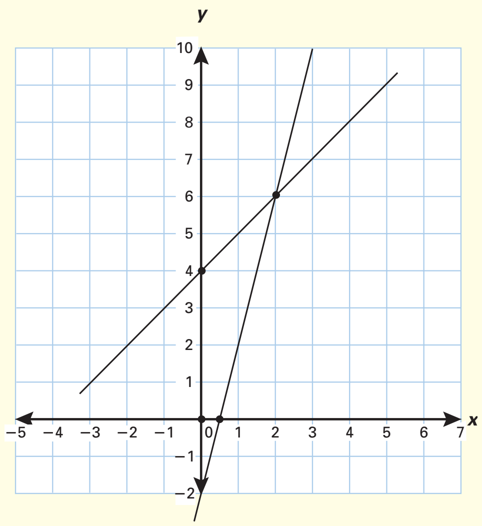
1. The point of intersection is $(-1, 3)$. This can be seen without a drawing because the x-coordinate must equal -1, and the y-coordinate must equal 3.

2. a



b. Any line with the same slope but a different y-intercept has no point of intersection with the given line, so the equation is
$y =$ *any number* $+ 4x$.

c. The point of intersection is (2, 6).



You can be sure that (2, 6) is correct by showing $x = 2$ and $y = 6$ fits both equations:

$6 = -2 + (4 \times 2)$
$6 = 4 + 2$

3. a. At the point of intersection, $x = -\frac{4}{5}$

$-3 - 2x = 1 + 3x$

$-3 = 1 + 5x$

$-4 = 5x$

$x = -\frac{4}{5}$

b. At the point of intersection, $y = -\frac{7}{5}$

$y = 1 + 3\left(-\frac{4}{5}\right)$

$y = \frac{5}{5} + -\frac{12}{5} = -\frac{7}{5}$

Hints and Comments

Materials

Student Activity Sheet 6 (one per student);
graph paper (one sheet per student).

Overview

Students read the Summary, which reviews the main concepts covered in this section. Students work on the Check Your Work and For Further Reflection problems to find the points of intersection of lines using graphic and algebraic methods. These problems are designed for student self-assessment. Students who have difficulties in answering the questions without help may need extra practice. This section is also useful for parents who want to help their children with their work. Answers are provided in the Student Book. Have students discuss their answers with a classmate.

About the Mathematics

Students may use several strategies for finding the point of intersection of two lines. It is important that they understand the concepts and procedures that play a role. Students should be able to solve these kinds of problems using paper and pencil.

E Intersecting Lines

Notes

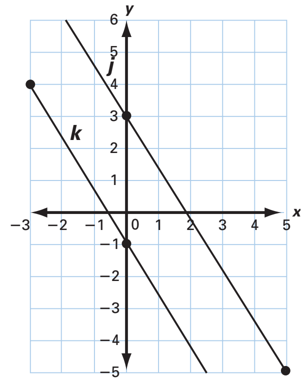
4 Students may need assistance finding appropriate points to use for finding the slopes of the lines, so they can then write the equations of the lines.

You may want to do this step as a class.

For Further Reflection

Reflective questions are meant to summarize and discuss important concepts.

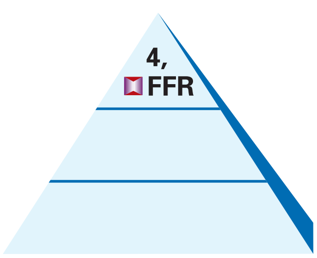
4. Write an equation for each line shown in the graph. Then use the equations to find the intersection of the two lines.



For Further Reflection

Graphs and equations can be used to describe lines and their intersections. Tell which is easier for you to use and explain why.

Assessment Pyramid



Assesses Section E Goals

Reaching All Learners

Intervention

Some students may need help with the fraction computation involved in solving problem 4. Students may need to be reminded to find a common denominator. Problem 4 is a multi-step question, so some students may benefit from thinking about and listing the needed steps.

Study Skills

The For Further Reflection question encourages metacognition—thinking about how you think and learn.

Solutions and Samples

4. Line j goes through points (0,3) and (5, −5), so the slope is $-\frac{8}{5}$. Line k goes through points (−3,4) and (0,−1), so the slope is $-\frac{5}{3}$.

The equation of line j is $y = 3 - \frac{8}{5}x$.

The equation of line k is $y = -1 - \frac{5}{3}x$.

The point of intersection is (−60, 99).
Sample strategy:

$3 - \frac{8}{5}x = -1 - \frac{5}{3}x$	add $\frac{8}{5}$ x to both sides
$3 = -1 + (\frac{8}{5} - \frac{5}{3})x$	add 1 to both sides
$4 = -\frac{1}{15}x$	multiply both sides by −15
$x = -60$	
$y = 3 - \frac{8}{5}(-60)$	
$y = 3 - (-96) = 99$	

For Further Reflection

Answers will vary according to student preferences.

Some students may find the use of graphs easier because they can see the lines and point(s) of intersection. Other students may find working with equations easier because they can read the slope and y-intercept immediately from the equation. Using the equation(s) they can find the point of intersection exactly even if the coordinates are not whole numbers. Students may describe different preferences for different situations or activities they have to do with the lines.

Hints and Comments

Overview

Students work on the Check Your Work and the For Further Reflection problems.

Additional Practice

Section A Where There's Smoke

Here is a map of the San Francisco Bay Area. There are seven airports located in this area. People in the control tower at each airport can see the control towers at the other airports.

Source: © Rand McNally.

Use degree measurements with 0° for north and measure in a clockwise direction to answer the following questions.

1. **a.** In what direction from the San Carlos airport is the Hayward airport?

   b. Looking from Oakland in the direction 335°, you can see the Alameda airport. What is the opposite direction of 335°? Which airport is approximately in that direction?

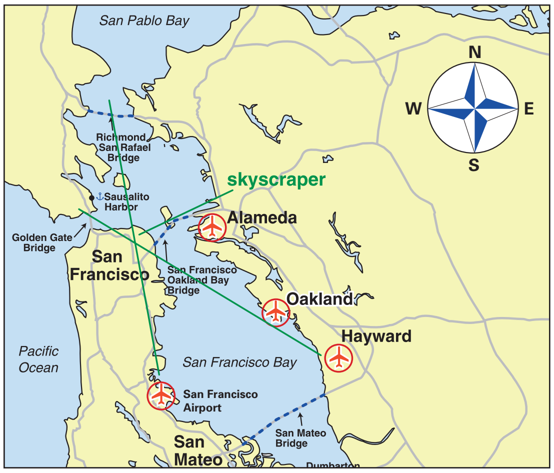
   c. From the Hayward airport, you can see a tall skyscraper in the direction 300°. This same skyscraper can be seen from the San Francisco airport in the direction 350°. Describe the location of this skyscraper on the map.

Section A. Where There's Smoke

1. **a.** Approximately 38°

   b. The opposite direction is 155°. Hayward airport is in that direction.

   c. The skyscraper is about 1 kilometer (km) west of the San Francisco-Oakland Bay Bridge. See the map.



Additional Practice

A grid has been put over the map of the San Francisco Bay Area. The seven airports in this area are marked with planes. The San Francisco airport has the coordinates (0, 0).

Source: © Rand McNally.

2. **a.** What are the coordinates of the Oakland airport?

   b. Sausalito Harbor is at coordinates (–4, 9). What is the equation of the line that is due north from Sausalito Harbor?

The San Mateo Bridge crosses the bay.

3. **a.** Use graph paper to draw the rectangular region that completely encloses the San Mateo Bridge. Use horizontal and vertical lines from the grid.

   b. Use inequalities to describe the region you drew in part **a**.

Section A. Where There's Smoke (continued)

2. a. The coordinates of the Oakland airport are (7, 4).

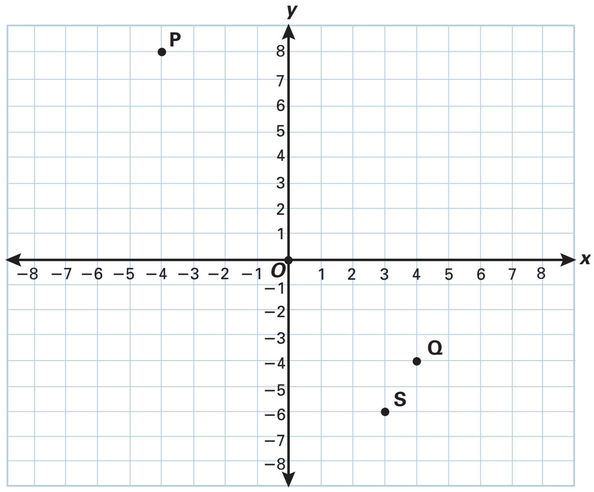
b. The equation is $x = -4$.

3. a. See the graph below. Students will have used graph paper without the map.

b. The region is described by the inequalities $5 < x < 10$ and $-3 < y < 0$

Additional Practice

Section B Directions as Pairs of Numbers



1. Describe point *P* on the graph as seen from the origin, using a pair of direction numbers.

2. Would point *Q* be on a line drawn through *O* and *P*? Explain why or why not.

3. Would point *S* be on a line drawn through *O* and *P*? Explain why or why not.

4. What is the slope of a line from point *Q* to point *P*?

Section C An Equation of a Line

1. **a.** On a sheet of graph paper, draw a line with a positive *y*-intercept and a negative slope. Call this line ℓ.

   b. What is the equation of line ℓ?

   c. What can you say about any line that is parallel to ℓ?

2. **a.** Draw a line with a negative *y*-intercept and a positive slope. Call this line *m*.

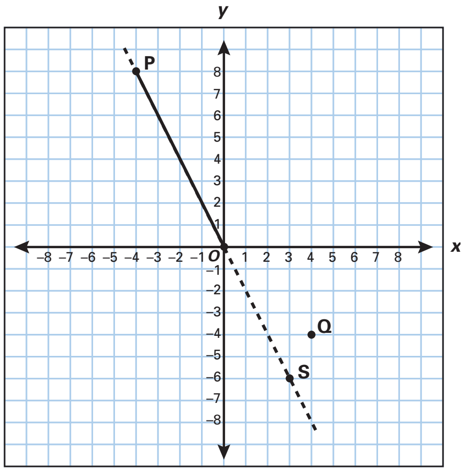
   b. Now draw a line that intersects line *m*. What is the slope of this line, and what is the intercept? What are the coordinates of the point of intersection of these two lines?

3. **a.** Draw a line whose equation is $y = 2x - 4$.

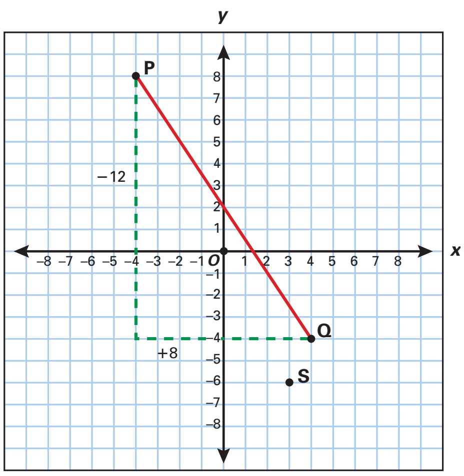
   b. What is the equation of the line that goes through (0, 0) and intersects the line $y = 2x - 4$ at the point (6, 8)?

Section B. Directions as Pairs of Numbers

1. P is in the direction [–4, 8].
2. No, Q is in the direction [4, –4]; that is a different direction than line OP which is [–4, 8].
3. Yes, S is on the same line, but in the opposite direction from O. See the graph below.

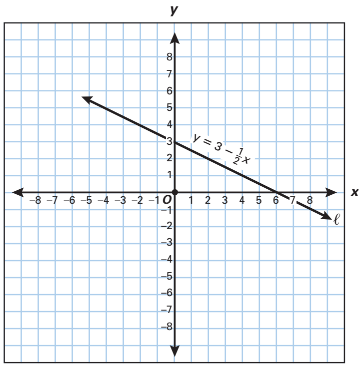


4. From P to Q, you go 12 units down and 8 to the right, so the slope is $\frac{-12}{8}$ or $-\frac{3}{2}$ or $-1\frac{1}{2}$. See drawing in graph.



Section C. Equations of a Line

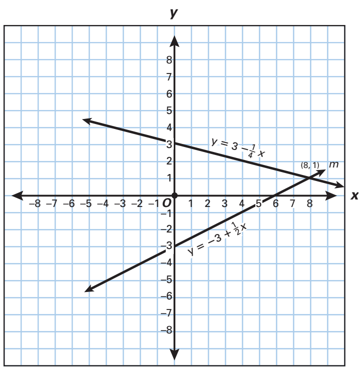
1. **a.** Student answers depend on their choice of line ℓ. Check that the y-intercept is positive and the slope is negative. Sample graph:



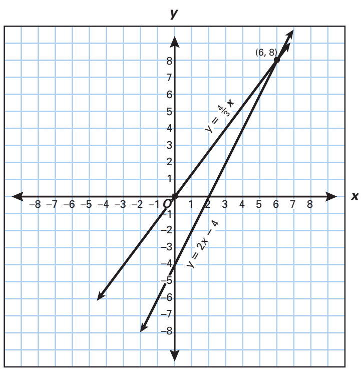
b. Student answers depend on their choice of line ℓ. Equation for sample graph: $y = 3 - \frac{1}{2}x$

c. A line that is parallel to line ℓ has the same slope. For the sample graph the slope is $-\frac{1}{2}$, so this new line would also have slope $-\frac{1}{2}$.

2. **a.** and **b.** Graphs will vary. Sample graphs:



3. **a.** and **b.**



An equation of the line through points (0, 0) and (6, 8) is $y = \frac{4}{3}x$ or an equivalent one. The y-intercept is (0, 0). The slope is equal to $\frac{8}{6}$ or $\frac{4}{3}$. So an equation is $y = \frac{4}{3}x$.

Additional Practice

Section D Solving Equations

1. Draw a diagram to illustrate each of the following equations. Then solve each equation.

   a. $12 + 2x = 5 + 4x$

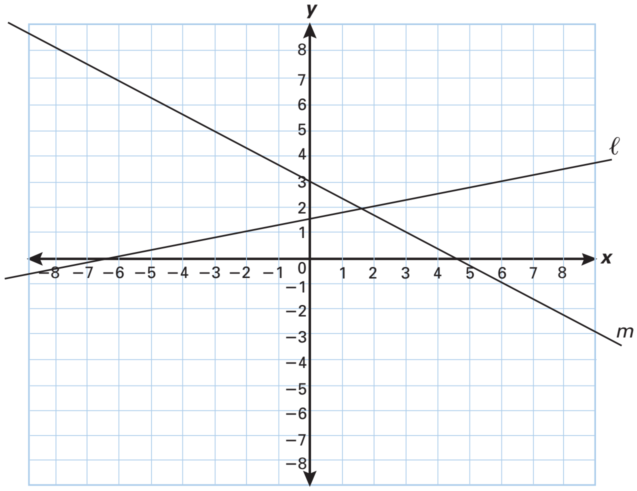
   b. $-5 + 3x = 16 - 4x$

2. Write a "frog problem" for each of the following equations. Then solve each equation.

   a. $4 + 3x = 19 + 2x$

   b. $-4 + 3x = -19 + 2x$

Section E Intersecting Lines



1. Which of the two lines shown on this graph has the equation $y = 1.5 + 0.25x$? Explain your answer.

2. What is the equation of the other line?

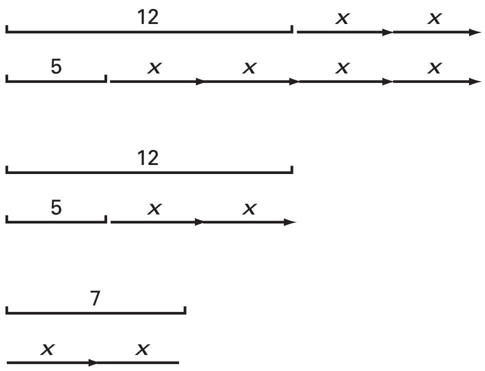
3. Find the point of intersection of the two lines.

4. Suppose you drew a line p that intersects line ℓ at (6, 3) and line m at (3, 1). What is the equation of line p?

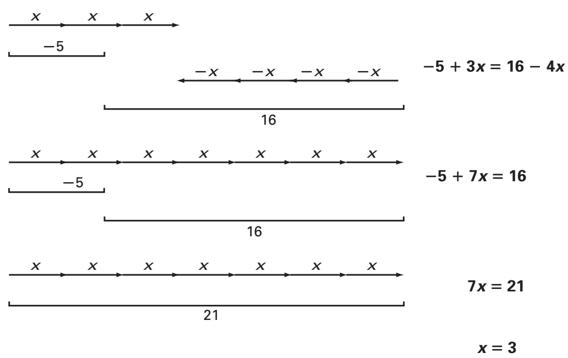
Section D. Solving Equations

1. a. $12 + 2x = 5 + 4x$

$x = 3.5$



b. $-5 + 3x = 16 - 4x$



2. a. Sample story problem:

Frog A starts at a distance of 4 and makes 3 jumps to the right. Frog B starts at a distance of 19 and makes 2 jumps to the right. The length of the jump x is 15.

$4 + 3x = 19 + 2x$

$4 + x = 19$

$x = 15$

b. Sample story problem: One frog starts at a distance 4 left of zero and makes three jumps to the right. The other frog starts at 19 left of zero and makes two jumps to the right. The length of the jump x is –15.

$-4 + 3x = -19 + 2x$

$-4 + x = -19$

$x = -15$

Section E. Intersecting Lines

1. Line ℓ, since it has a positive slope and the y-intercept is 1.5

2. Line m has equation $y = 3 - \frac{4}{6}x$ or $y = 3 - \frac{2}{3}x$ or an equivalent one.

3. Point of intersection: $(\frac{18}{11}, \frac{21}{11})$.
Sample student work:

$\frac{3}{2} + \frac{1}{4}x = 3 - \frac{2}{3}x$

$\frac{3}{2} + \frac{3}{12}x = 3 - \frac{8}{12}x$

$\frac{3}{12}x + \frac{8}{12}x = 3 - \frac{3}{2}$

$\frac{11}{12}x = \frac{3}{2}$

$12 \times \frac{11}{12}x = 12 \times \frac{3}{2}$

$11x = 18$

$x = \frac{18}{11}$

If $x = \frac{18}{11}$, $y = \frac{3}{2} + (\frac{1}{4} \times \frac{18}{11})$
(or $y = 3 - (\frac{2}{3} \times \frac{18}{11})$).

$y = \frac{3}{2} + \frac{18}{44}$ or $y = \frac{3}{2} + \frac{9}{22}$

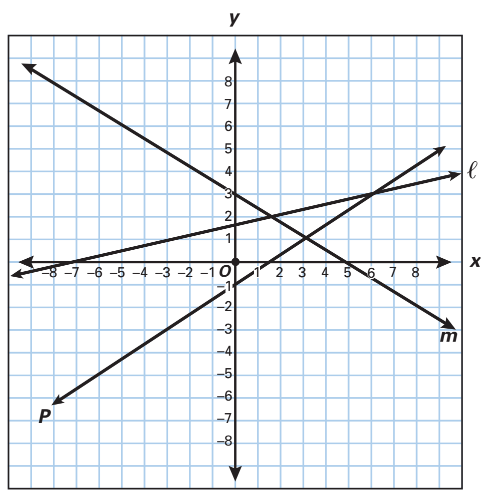
$y = \frac{33}{22} + \frac{9}{22}$

$y = \frac{42}{22}$

$y = \frac{21}{11}$

The point of intersection is $(\frac{18}{11}, \frac{21}{11})$.

4. The equation of line p is $y = -1 + \frac{2}{3}x$.



Assessment Overview

Unit assessments in *Mathematics in Context* include two quizzes and a Unit Test. Quiz 1 is to be used anytime after students have completed Section B. Quiz 2 can be used after students have completed Section D. The Unit Test addresses most of the major goals of the unit. You can evaluate student responses to these assessments to determine what each student knows about the content goals addressed in this unit.

Pacing

Each quiz is designed to take approximately 25 minutes to complete. The Unit Test was designed to be completed during a 45-minute class period. For more information on how to use these assessments, see the Planning Assessment section on the next page.

Goals	Assessment Opportunities		Problem Levels
• Describe and graph directions using compass directions, angles, and direction pairs.	Quiz 1 Test	Problems 1abdce, 3abcd, 4a Problems 1ab	Level I
• Understand and graph horizontal and vertical lines and their equations.	Quiz 1	Problems 2c, 3d	Level I
• Find and use equations of the form $y = i + sx$ using the slope and y-intercept.	Quiz 2 Test	Problems 1bcd, 2cd Problems 5ab	Level I
• Graph points and lines in a coordinate system.	Quiz 1 Quiz 2 Test	Problems 2abc, 3bc, 4a Problems 1ab, 2ab Problems 2ab, 3abd	Level I
• Solve equations of the form $a + bx = c + dx$.	Quiz 2 Test	Problem 3 Problems 4abcd	Level I
• Understand the meaning of slope in different contexts.	Quiz 1 Quiz 2 Test	Problem 4b Problem 1e Problem 3e	Level II
• Understand how to find the intersection point of two lines, algebraically and graphically.	Test	Problems 2cd, 3c	Level II
• Understand the graph of a line in the coordinate plane.	Quiz 2 Test	Problem 1f Problem 3d	Level II
• Choose an appropriate way to solve equations.	Test	Problem 5d	Level III

About the Mathematics

These assessment activities assess the major goals of the *Graphing Equations* unit. Refer to the Goals and Assessment Opportunities section on the previous page for information regarding the goals that are assessed in each problem. Some of the problems that involve multiple skills and processes address more than one unit goal. To assess students' ability to engage in non-routine problem solving (a Level III goal in the Assessment Pyramid), some problems assess students' ability to use their skills and conceptual knowledge in new situations.

Planning Assessment

These assessments are designed for individual assessment; however, some problems can be done in pairs or small groups. It is important that students work individually if you want to evaluate each student's understanding and abilities.

Make sure you allow enough time for students to complete the problems. If students need more than one class session to complete the problems, it is suggested that they finish during the next mathematics class, or you may assign select problems as a take-home activity. Students should be free to solve the problems their own way. Calculators may be used on the quizzes or Unit Test if students choose to use them.

If individual students have difficulty with any particular problems, you may give the student the option of making a second attempt after providing him or her a hint. You may also decide to use one of the optional problems or Extension activities not previously done in class as additional assessments for students who need additional help.

Scoring

Solution and scoring guides are included for each quiz and the Unit Test. The method of scoring depends on the types of questions on each assessment. A holistic scoring approach could also be used to evaluate an entire quiz.

Several problems require students to explain their reasoning or justify their answers. For these questions, the reasoning used by students in solving the problems as well as the correctness of the answers should be considered in your scoring and grading scheme.

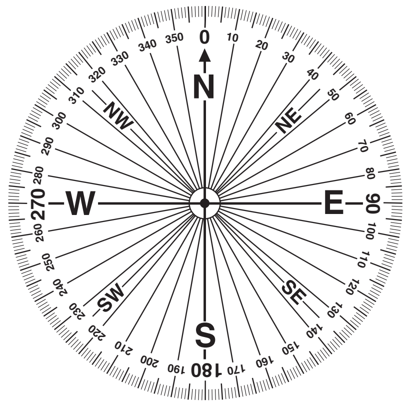
Student progress toward goals of the unit should be considered when reviewing student work. Descriptive statements and specific feedback are often more informative to students than a total score or grade. You might choose to record descriptive statements of select aspects of student work as evidence of student progress toward specific goals of the unit that you have identified as essential.

Name ______________________ Date ______________

Graphing Equations Quiz 1

Use additional paper as needed.

1. Use the compass card on the right to answer these questions.



a. What degree measurement corresponds to the direction NE?

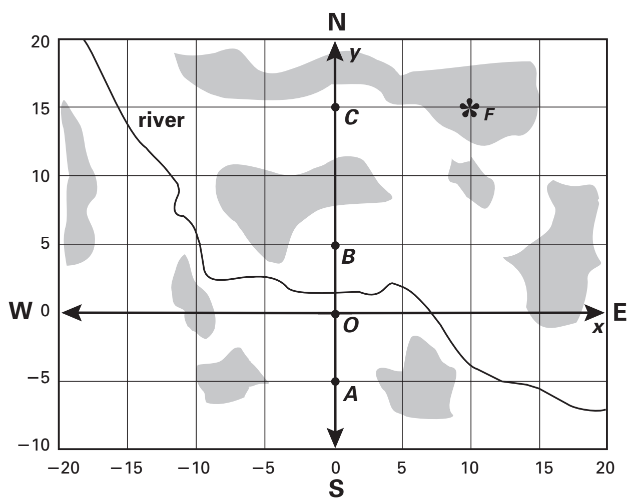
b. What degree measurement corresponds to SE?

c. What compass direction is opposite NE?

d. What compass direction is opposite SE?

e. What degree measurement corresponds to the opposite of SE?

2. The numbers on the screen on the right represent distances in kilometers. What are the coordinates of the points that are:



a. 15 km east of A?

b. 15 km west of A?

c. On the screen draw the line described by $x = -15$.

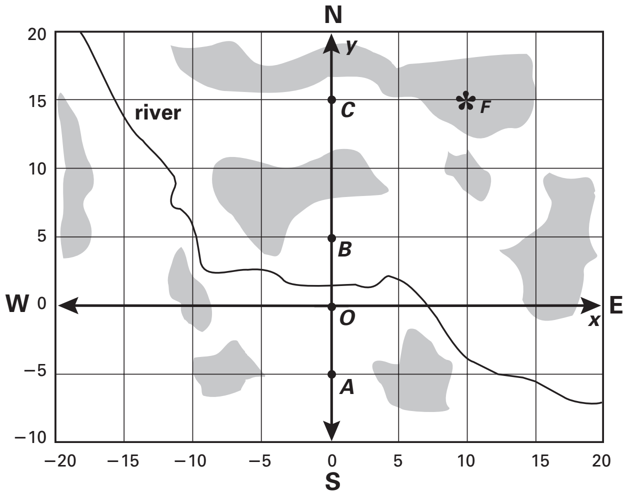
3. A fire is spotted at point G (16, 1).

a. What directions, measured in degrees, should be given to the firefighters at towers A, B, and C?

A: ___________ B: ___________ C: ___________

b. Write direction pairs to describe the direction of the fire at point G as seen from points A, B, and C.

A: ___________ B: ___________ C: ___________

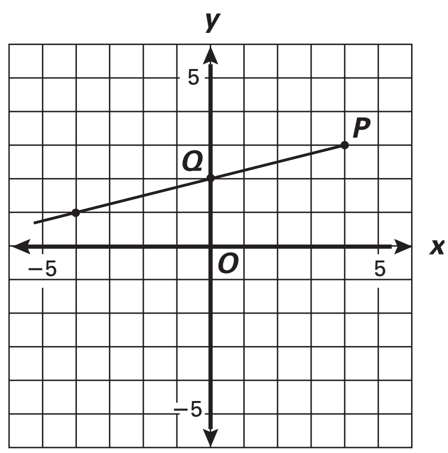


A strong wind from the south blows the fire to point H, which is 4 km north of G.

c. What are the coordinates of H?

d. Give a direction pair that indicates the direction of H as seen from G.

4. a. Describe the direction from point P to Q using a direction pair.



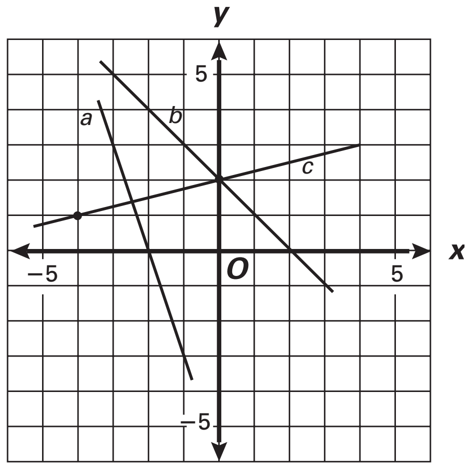
b. What is the slope of the line through P and Q?

Name ______________________ Date ______________

Graphing Equations Quiz 2

Use additional paper as needed.

1. Lines *a, b,* and *c* are drawn in the coordinate plane on the right.



   a. Which two lines have the same *y*-intercept?

   b. What is the *y*-intercept for these two lines?

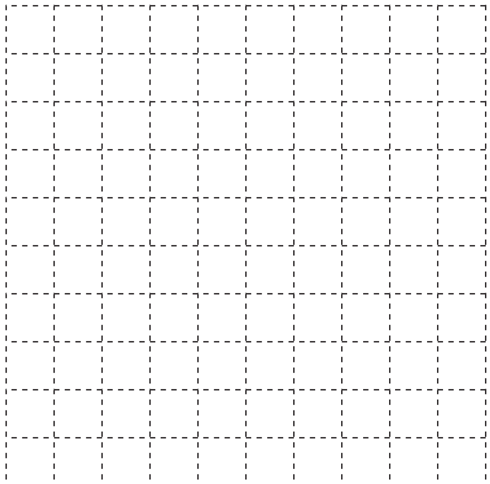
   c. Which line(s) have positive slopes?

   d. Which line(s) have negative slopes?

   e. Explain how you decided if a line had a positive or negative slope.

   f. What is the *y*-intercept of line *a*? Explain how you found your answer.

2. a. Draw a coordinate system on the grid below. Be sure to include all necessary labels.



b. Draw a line through points *D* (0,2) and *E* (3,4).

c. What is the slope of this line? (Show how you found your answer.)

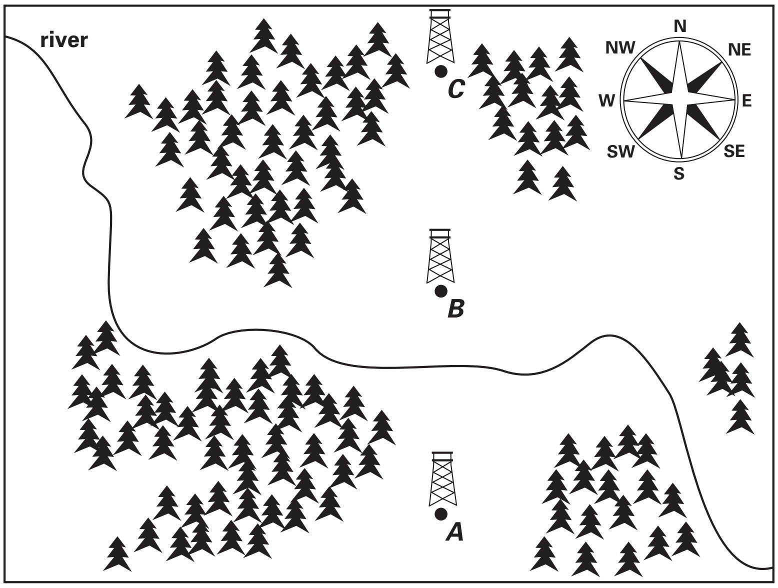
d. Write the equation for this line.

3. Solve the following equation. (Show your work.)

$15 + 5u = 21 + 3u$

Use additional paper as needed.

1.



Firefighters receive reports of smoke spotted at:

294° from tower *A*

247° from tower *B*

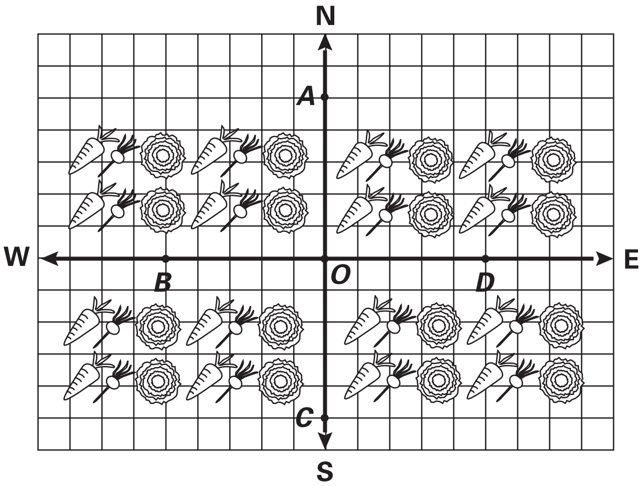
210° from tower *C*

a. The firefighters know something is wrong with these reports. Explain how they know.

b. Further reports confirm that the observations from towers *A* and *C* are correct but the observation from tower *B* is incorrect. What should be reported from tower *B*?

2. Grandma Bessy loves to eat fresh vegetables from her own garden. However, there are others who also like Grandma's vegetables: the snails in her garden. So Grandma Bessy is always trying to catch the snails. She even built a watchtower in the center of the garden to locate snails. Her grandchildren help her hunt for snails....

Grandma Bessy's garden is shown in the coordinate system on the right. She built her watchtower at the origin, *O*. Her four grandchildren, Annie, Beth, Carlos, and David, are standing at points *A* (0 ,5), *B* (−5, 0), *C* (0, −5), and *D* (5, 0), respectively. They are all 5 m away from Grandma Bessy. As soon as Grandma spots a snail, she directs one of her grandchildren to it.



Map of Grandma Bessy's garden.

Grandma spots a snail. She tells David to go in direction [+1, +4]. She tells Annie to proceed in direction [+6, −1].

a. What are the coordinates of the snail she found?

Grandma tells Carlos to go east and David to go south.

b. What are the coordinates of this snail?

Grandma spots a snail at (−1, 1). She needs to direct two of her grandchildren to go there.

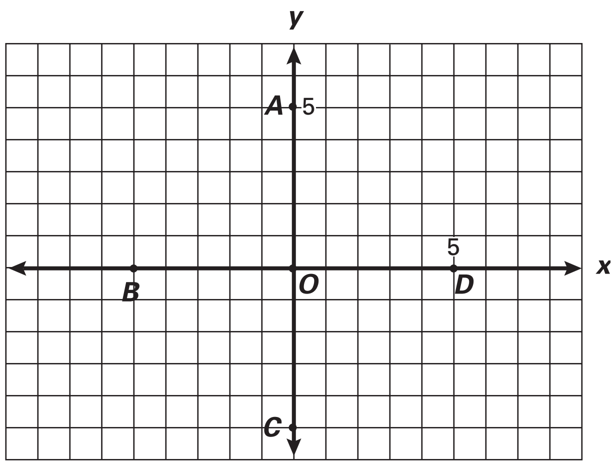
c. Whom should she direct?

d. Using compass directions (in degrees), what instructions should she give?

3. Grandma Bessy's garden is shown on the right as a coordinate system with *x*- and *y*-axes.

Annie and Beth draw a line connecting points *A* and *B*. On this line, they put salt to catch the snails.

a. Draw this line and write an equation for the line through *A* and *B*.



Grandma sees a straight line of snails, in single file formation, moving slowly forward in direction [−2, +2]. The lead snail is now at point (0, 2).

b. Draw this line of snails and write the equation for the line.

c. At what point will the lead snail cross the line of salt?

d. In what direction should the snails travel to avoid the line of salt? Explain your reasoning and illustrate on the graph.

Another straight line of snails is moving on a line with slope $\frac{2}{3}$. This line goes through David's location, (5, 0).

e. What is the *y*-intercept of this line? Show how you found your answer.

4. Solve the following equations:

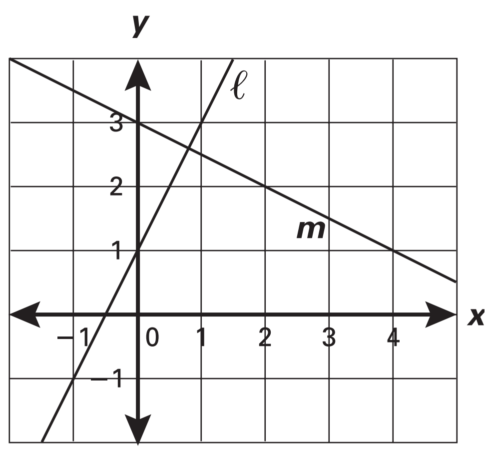
a. $-6 + 5x = -21 + 2x$

b. $-1 + 3x = -1 - x$

c. $7x = 25 + 2x$

d. $15 + 5x = 0$

5. Line m is the graph of $y = 3 - 0.5x$.



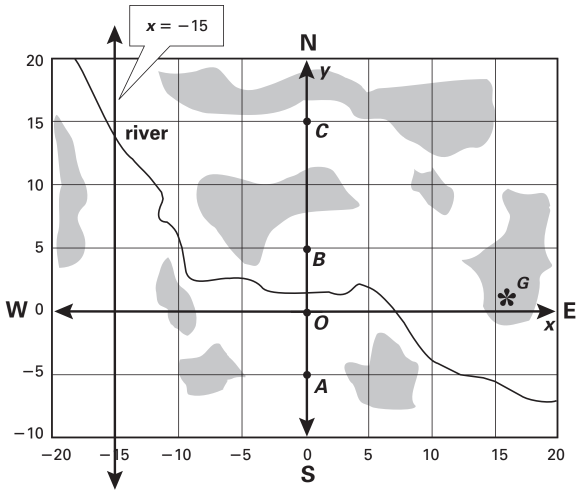
a. What information does the number 3 give you for line m?

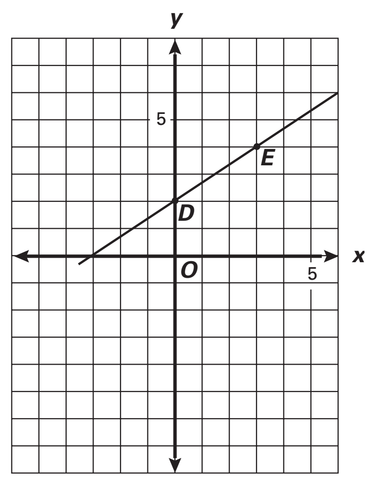
b. What information does the number -0.5 give you for line m?

c. What is the equation for line ℓ ?

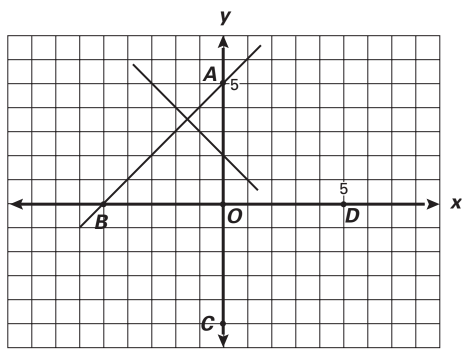
d. Find the coordinates of the point of intersection for lines ℓ and m.

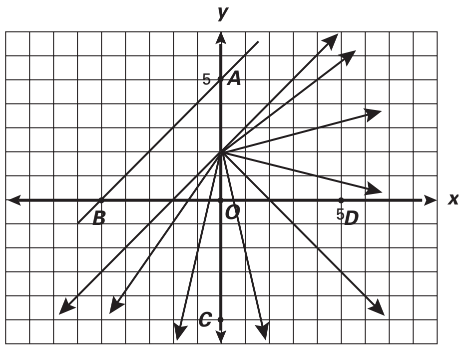
Graphing Equations Quiz 1
Solution and Scoring Guide

Possible student answer	Suggested number of score points	Problem level
1. a. 45°	**1**	**I**
b. 135°	**1**	**I**
c. SW	**1**	**I**
d. NW	**1**	**I**
e. 315° (NW)	**1**	**I**
2. a. (15,−5)	**1**	**I**
b. (−15,−5)	**1**	**I**
c. 	**1**	**I**
3. a. A: 70°, B: 105°, C: 130°	**3**	**I**
b. A: [+16, +6], B: [+16, −4], C: [+16, −14]	**3**	**I**
c. (16, 5)	**1**	**I**
d. [0, +4]	**1**	**I**
4. a. [−4, −1]	**1**	**I**
b. slope = $\frac{1}{4}$	**1**	**II**
Total score points	**18**	

Possible student answer	Suggested number of score points	Problem level
1. a. lines *b* and *c*	**1**	**I**
b. 2 or (0,2)	**1**	**I**
c. line *c*	**1**	**I**
d. lines *a* and *b*	**1**	**I**
e. Possible explanation: Following the lines from left to right, *a* and *b* "go down" and *c* "goes up."	**1**	**II**
f. -6; each step to the right, the graph goes 3 steps down; one point of the line is $(-1, -3)$, so $(0, -6)$ is also on the line.	**2** (Award 1 point for the explanation.)	**II**
2. a. Subtract 1 point if the axes, origin, and scale are not labeled.	**3**	**I**
 **b.** A line should be drawn through points *D* and *E*, as shown on the left.	**2** (Award 1 point for each.)	**I**
c. The slope is $\frac{2}{3}$. The slope can be found in different ways; one way is to use the components of the direction pair from *D* to *E*; it can also be illustrated in the graph.	**2**	**I**
d. $y = 2 + \frac{2}{3}x$ or $y = \frac{2}{3}x + 2$ (Do not subtract points if an incorrect answer for part **c** is used for the slope.)	**2**	**I**
3. $u = 3$ Solution: $15 + 5u = 21 + 3u$ $15 + 2u = 21$ $2u = 6$ $u = 3$	**2**	**I**
Total score points	**18**	

Graphing Equations Unit Test Solution and Scoring Guide

Possible student answer	Suggested number of score points	Problem level
1. a. The three directions do not determine a single point, so at least one report must be wrong.	**2**	**I**
b. Tower B's direction should have been about 238°. This direction can be determined by drawing a line from the intersection of the lines from tower A and tower C to tower B.	**2**	**I**
2. a. (6, 4).	**1**	**I**
b. (5, −5)	**1**	**I**
c. Annie and Beth (they are the closest)	**2**	**II**
d. Annie should go 195°; Beth should go 75°. (Other combinations of two grandchildren can be given full credit. For example, David should go 280°, and Carlos should go 350°.)	**2**	**II**
3. a. $y = 5 + x$	**1**	**I**
b. $y = 2 - x$	**2**	**I**
c. (−1.5, 3.5). This can be estimated from the graph, but should be checked by solving. 	**2**	**II**

Possible student answer	Suggested number of score points	Problem level
d.  Any direction below the line parallel to the line of salt (*AB*). This can also be given in degrees (between 45° and 135°), or compass directions (from NE to SW). Some examples of correct answers given as direction pairs include: [+2, +2] or [−2, −2] (parallel to the salty line), [+2, +1.5], [+2, +1.4], [+2, 0], [+2, −0.5], [+2, −2] (opposite the initial direction), and many others. Equations of lines are not appropriate since lines travel in two directions.	**3**	**I/II**
e. $-3\frac{1}{3}$; Possible explanation: Each time you move to the left from point *D*, the line moves down $\frac{2}{3}$. Since $\frac{2}{3} \times 5 = \frac{10}{3}$, the y-intercept will be $\frac{10}{3}$ below the x-axis, which is $-3\frac{1}{3}$.	**2**	**II**
4. a. $x = -5$ **b.** $x = 0$ **c.** $x = 5$ **d.** $x = -3$	**2** **2** **2** **2**	**I** **I** **I** **I**
5. a. y-intercept of the line **b.** the slope of the line **c.** $y = 1 + 2x$ **d.** $1 + 2x = 3 - 0.5x$ $2x = 2 - 0.5x$ $2.5x = 2$ $x = 0.8$; $y = 2.6$	**1** **1** **1** **2**	**I** **I** **I** **II/III**
Total score points	**33**	

Glossary

The Glossary defines all vocabulary words indicated in this unit. It includes the mathematical terms that may be new to students, as well as words having to do with the contexts introduced in the unit. (Note: The Student Book has no Glossary. Instead, students are encouraged to construct their own definitions, based on their personal experiences with the unit activities.)

The definitions below are specific for the use of the terms in this unit. The page numbers given are from the Student Book.

< and > signs (p. 7) inequality symbols that represent the relations less than (<) and greater than (>).

compass rose (p. 2) a circle that shows the four main directions (north, east, south, and west) and the in-between directions (northeast, southeast, southwest, and northwest).

coordinate system (p. 4) a rectangular grid used as a means for locating points in a plane; such a system has two number lines (called *axes*) placed at right angles to one another, which intersect at a point called the origin.

degree measurements (p. 2) directions as measured in degrees; a circle has 360° (north can be placed at 0 degrees and measurements can be taken clockwise, or the positive x-axis can be 0 degrees and measurements can be taken counterclockwise).

direction pair (p. 11) a direction pair is a pair of numbers that indicates a direction in a grid. The first number gives the horizontal component of the direction and the second number gives the vertical component. The numbers are preceded by a + sign or a − minus sign indicating to the right or up and to the left or down. Direction pairs are enclosed in brackets [,] to distinguish them from coordinate pairs.

equation of a line (p. 22) an equation such as $y = a + bx$ that when graphed is a line. An equation is a formula expressed as an equivalency ($y = 3$).

horizontal line (p. 6) a fixed line in a coordinate system parallel to the x-axis, with an equation of the form $y = a$

horizontal component (p. 3) the distance in the horizontal direction that is used to calculate the slope of a line

inequality (p. 6) algebraic notation involving < or > signs to describe a region bounded by one or more lines

intercept (p. 26) the point at which a graph intersects the x- or y-axis

origin (p. 4) the point where the x- and y-axes intersect in a coordinate system

quadrant (p. 4) one of four parts of a coordinate system as defined by the x- and y-axes. In this unit the quadrants are indicated by their wind directions NE, SE, SW and NW.

slope (p. 15) a measure of the steepness and direction of a line; the slope can be found by using two points on the line; it is the ratio of the vertical distance between the two points divided by the horizontal distance between the same two points. Instead of two points students may want to use the ratio of the components of a direction pair for the line.

tangent ratio (p. 24) a measure of the steepness of a line; the tangent of the angle that a line makes with the right side of the x-axis is the slope of the line, but only if the axes are both scaled in the same way.

unknown (p. 30) a number that is not known, which is usually represented by a letter or a symbol in an equation or inequality

vertical line (p. 5) a fixed line in a coordinate system parallel to the y-axis, with an equation in the form $x = b$

vertical component (p. 3) the distance in the vertical direction that is used to measure the slope of a line

***x*-axis** (p. 4) the horizontal axis in a rectangular coordinate system.

***x*-coordinate** (p. 3) a number that designates the distance along the horizontal axis.

***y*-axis** (p. 4) the vertical axis in a coordinate system.

***y*-coordinate** (p. 3) a number that designates the distance along the vertical axis.

***y*-intercept of a line** (p. 23) the height where a line crosses the y-axis or the y-coordinate of the point of intersection of the line and the y-axis

BRITANNICA

Mathematics in Context

Blackline Masters

Letter to the Family

Dear Family,

Your child will soon begin the *Mathematics in Context* unit *Graphing Equations.* Below is a letter to your child that opens the unit, describing the unit and its goals.

Your child will look at the way forest fires are reported from two lookout towers. This leads to a number of different ways of describing the location of the fire. Your child will describe locations of fires, first based on directions and then using more mathematical methods, such as using equations in a coordinate plane and solving systems of equations. In the context of a realistic situation involving a fire, your child will solve some complex mathematical equations. You might ask your child to share what he or she is learning about locating fires.

In this unit, your child will use a graphing calculator as a tool to graph lines and find an equation for a drawn line. Algebra will also be connected to geometry, by exploring the relationship between slope and the tangent ratio.

Toward the end of the unit, you might ask your child how he or she can find the intersection of one or more lines by using graphs and solving equations.

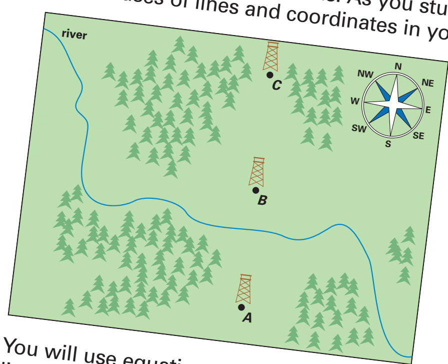
We hope you enjoy helping your child explore linear equations.

Sincerely,

The Mathematics in Context Development Team

Dear Student,

Graphing Equations is about the study of lines and solving equations. At first you will investigate how park rangers at observation towers report forest fires. You will learn many different ways to describe directions, lines, and locations. As you study the unit, look around you for uses of lines and coordinates in your day-to-day activities.



You will use equations and inequalities as a compact way to describe lines and regions.

A “frog” will help you solve equations by jumping on a number line. You will learn that some equations can also be solved by drawing the lines they represent and finding out where they intersect.

We hope you will enjoy this unit.

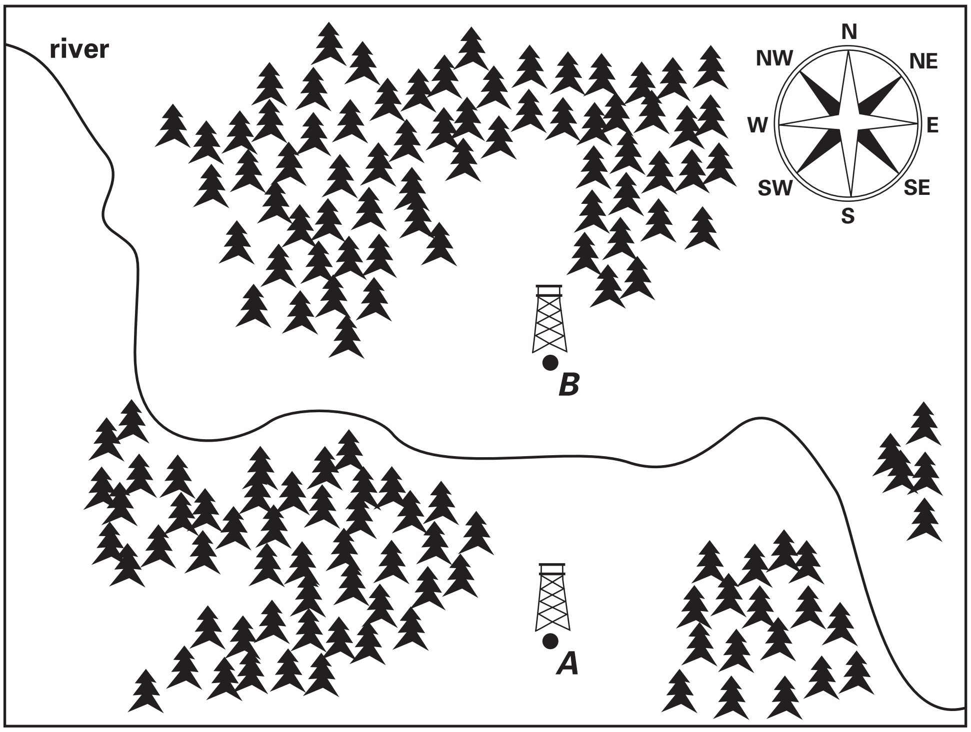
Sincerely,

The Mathematics in Context Development Team

Name ____________________

Student Activity Sheet 1

Use with *Graphing Equations*, page 2.



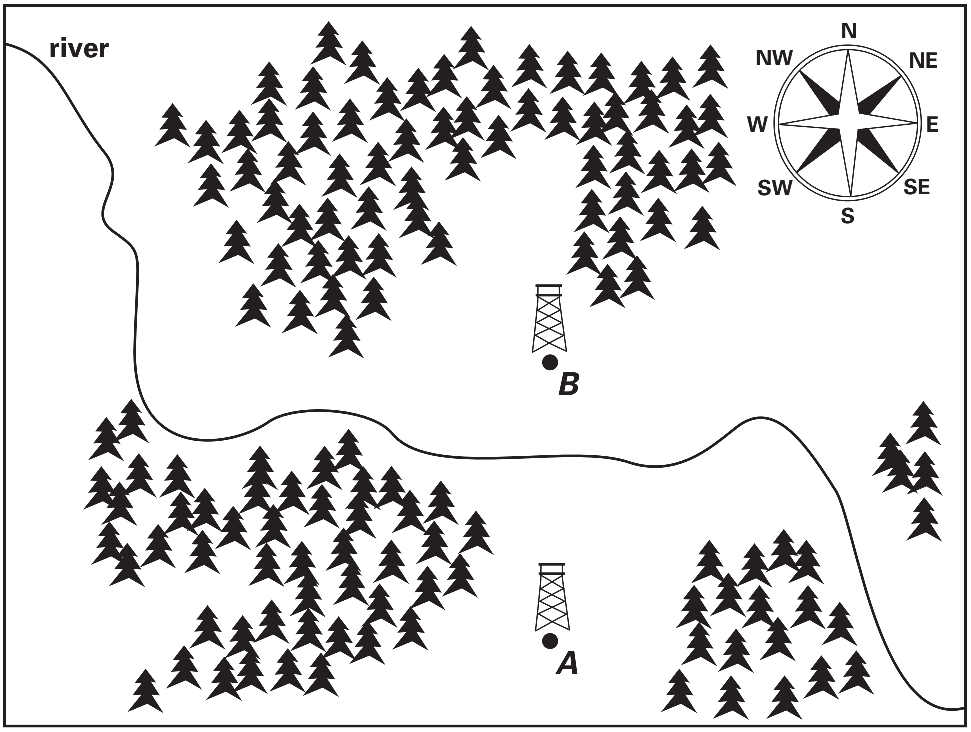
Student Activity Sheet 2

Name ____________________________________

Use with *Graphing Equations*, page 3.

3. Smoke is reported at 8° from tower *A*, and the same smoke is reported at 26° from tower *B*. Use this sheet to show the exact location of the fire.

4. Use this sheet to show the exact location of a fire if rangers report smoke at 342° from tower *A* and 315° from tower *B*.



Name ______________________________________

Student Activity Sheet 3

Use with *Graphing Equations*, pages 6 and 7.

15. Suppose some firebreaks follow parts of the lines described by the equations $x = 14$, $x = 16$, $x = 18$, $y = 8$, $y = 6$, $y = 4$, $y = 2$, and $y = 0$.

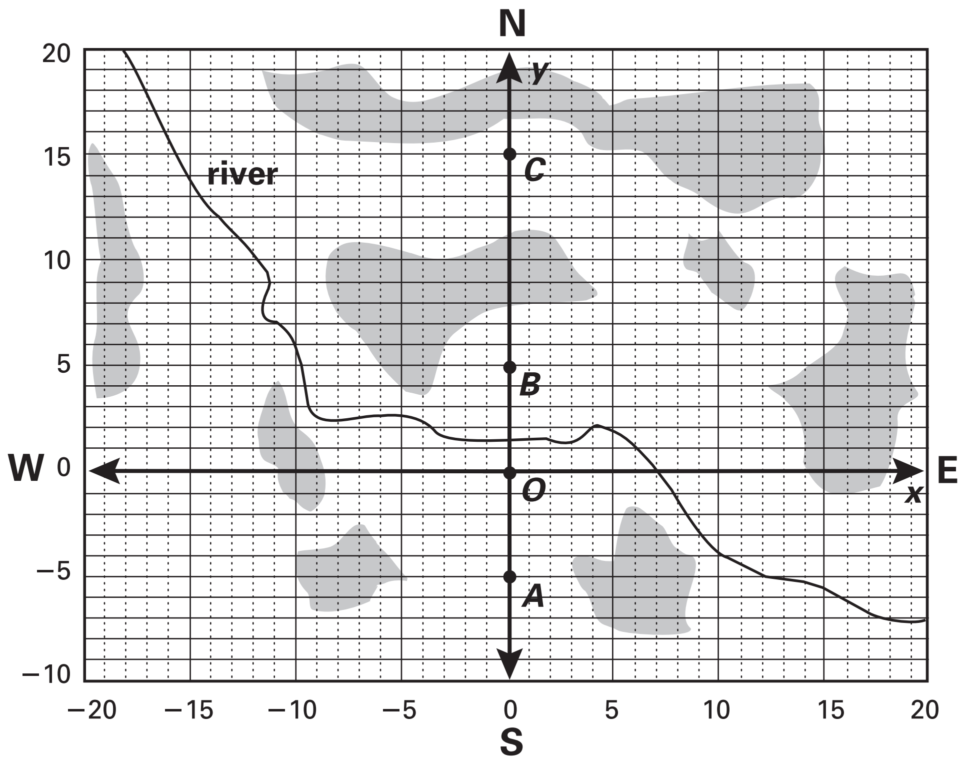
a. Draw the firebreaks through the wooded regions of the park on the graph below.

b. Write down the coordinates of 5 points that lie north of the firebreak described by $y = 8$.

17. Show the restricted region for a fire that starts at the point (17,5).

18. Another fire starts at the point (15,3). The fire is restricted to a region by four firebreaks. Show the region on the graph and describe it.

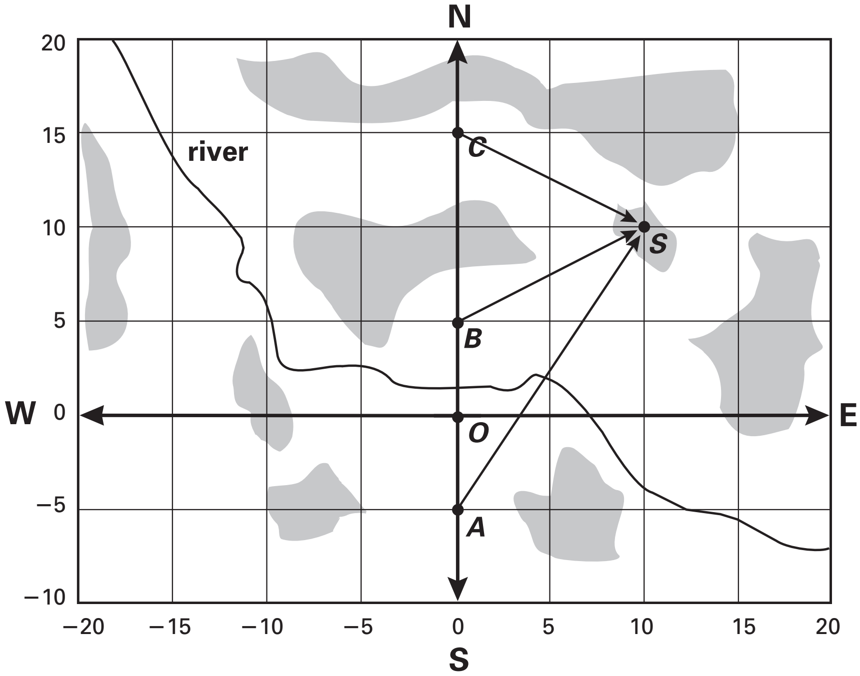
19. Use this graph and a pencil of a different color to show the region described by the inequalities $-6 < x < -3$ and $6 < y < 10$.

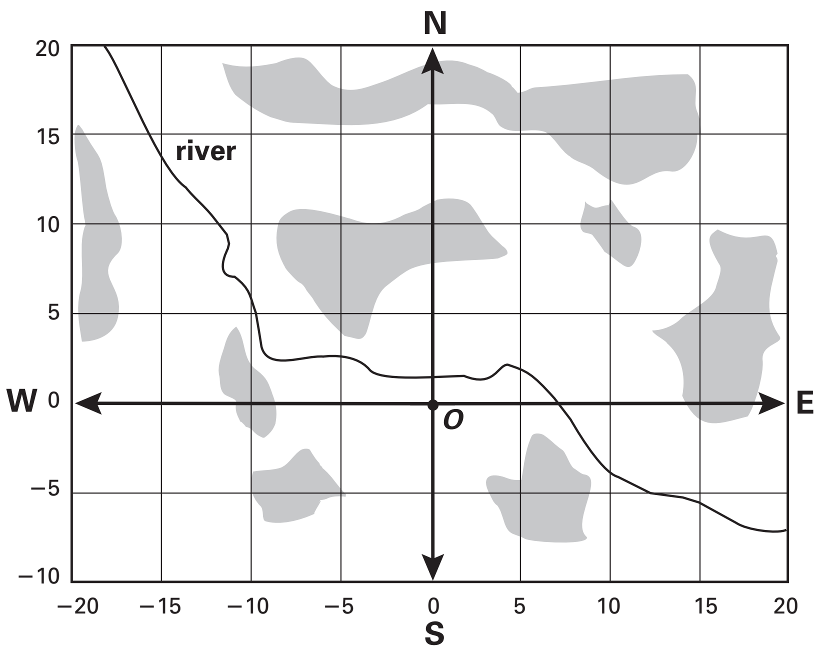


Student Activity Sheet 4

Use with *Graphing Equations*, pages 12–14.

Name ______________________________





Each of the lines drawn below contains the point (0, 0). For some of the lines, the slope is labeled inside its corresponding circle.

17. a. Fill in the empty circles with the correct slope.

b. What is the slope for a line that goes through the points (1, 1) and (15, 3)? How did you find out?

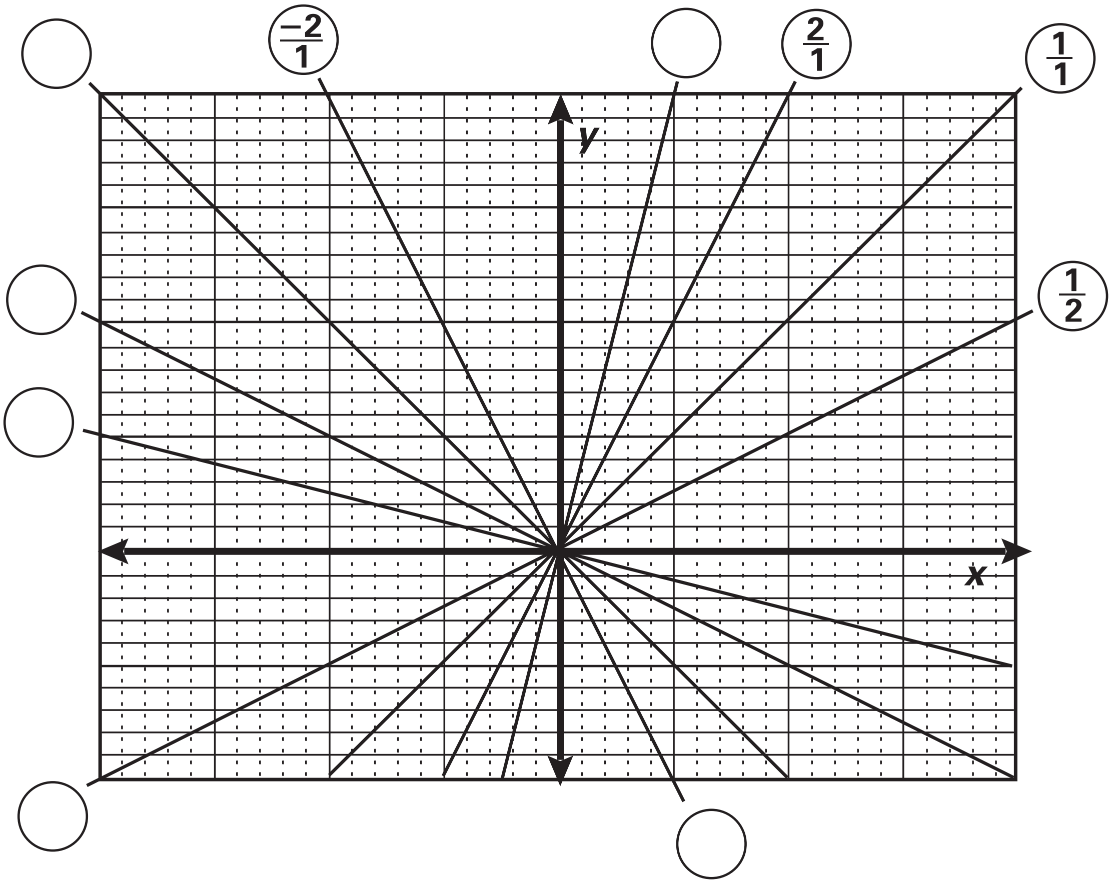
18. a. What do you know about two lines that have the same slope?

b. Explain that $\frac{3}{1}$, $\frac{6}{2}$, $\frac{-3}{-1}$, and $\frac{15}{5}$ all indicate the same slope. What is the simplest way to write this slope?

19. Draw and label the line through (0,0) whose slope is:

a. $\frac{4}{3}$

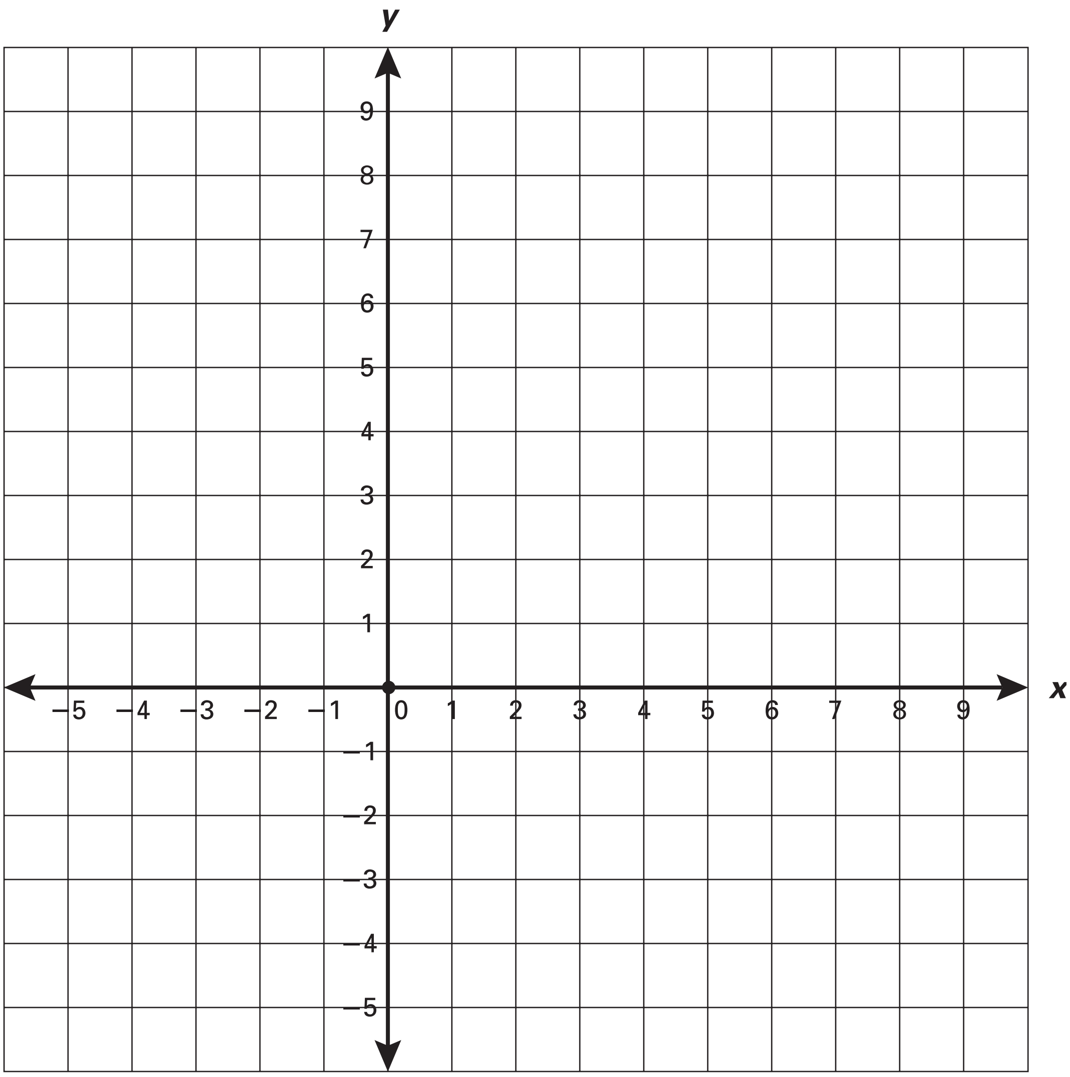
b. $-\frac{1}{2}$



Student Activity Sheet 6

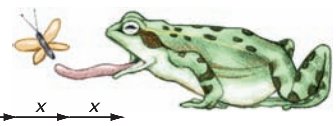
Name ______________________________

Use with *Graphing Equations*, page 40.



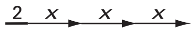
Solving Equations D

8. Write a "frog problem" and an expression to represent each diagram in parts **a** and **b**.



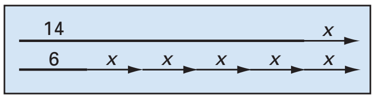
a.

b.



9. **a.** Write a "frog problem" and an equation to represent the diagram in Box A below.

Box A



b. Draw the diagram for the next step in finding the value of x and write the equation for your diagram.

c. Complete the sequence of diagrams and equations.

10. Here is an equation: $12 + 2x = 6 + 4x$.

a. Use a sequence of boxes to solve the equation. Start by drawing a diagram to represent each side of the equation.

b. Draw the rest of the boxes and diagrams to solve the equation.

11. **a.** Describe the equation $11 + 9x = 26 + 4x$ as a "frog problem."

b. Find the value of x in the equation and explain the steps in your solution. You may want to use a series of boxes, diagrams, and equations as part of your explanation.

12. Solve each equation for its unknown value and explain your method. How can you be sure that your answers are correct?

a. $100 + w + w = 75 + w + w + w + w$

b. $y + 42 + y = 12 + 3y + 2y$

c. $144 + z = 120 + 9z$

Hints and Comments
(continued from page 31T)

Comments About the Solutions

8. Note that rather than do a "frog problem," students have to write a "frog story" for each diagram. Students can make these stories into problems by choosing a number for the final distance and asking for the length of the jump that results in their final distance. For example, for problem 8a they may say, *If the final distance is 22, how long is each jump?* or *Find x for* $10 + 4x = 22$.

9. Problem 9 combines two equations and is similar to problems 4–7. Although the lengths 6, 14, and x are drawn to scale, students do not necessarily need to make their drawings to scale as well.

When students understand that the boxes and diagrams are sketches to help them visualize the equation, they should realize that the scale is not important. After students complete the problem, they should start to understand how to solve an equation using box diagrams and equations to describe the steps of the process.

10. and **11.**
When students feel comfortable solving the problems without drawing the boxes, let them do so, but make sure they can explain each step of their solutions.

12. Students should recognize that these problems are formulated in a slightly different way from the previous problems.

a. Students should recognize that $w + w = 2w$.

b. Students should recognize that $y + 42 + y = 42 + 2y$ and that $3y + 2y = 5y$.

Intersecting Lines

The park supervisor has just received two messages:

Smoke is reported on the line $y = 15 - x$.

Smoke is reported on the line $y = 5 + 4x$.

4. a. Which tower sent each message?

b. Calculate the coordinates of the smoke.

5. Repeat problem 4 for these two messages:

Smoke is reported on the line $y = 5 + x$.

Smoke is reported on the line $y = -5 + 1\frac{1}{4}x$.

The park supervisor received the message $y = 15 + 2x$ from tower C and the message $y = 5 + 3x$ from tower B.

6. What message do you expect from tower A?

7. Make up your own set of messages and find the location they describe.

Use **Student Activity Sheet 6** for problems 8 and 9.

8. a. Draw the line $y = 5$; label it l. Draw the line $y = -3 + 2x$ and label it m in the coordinate system.

b. Find the point of intersection of the two lines on the graph; write down the coordinates.

c. Use the equations of the lines to check whether the coordinates you found in **b** are correct.

9. a. In the same coordinate system you used for problem 8, draw the line $y = 4 - 2x$; label it n.

b. Estimate the coordinates of the point of intersection of lines m and n.

c. Solve the equation $-3 + 2x = 4 - 2x$.

d. Are your answers for **b** and **c** the same? Explain why or why not.

Hints and Comments
(continued from page 40T)

Materials

Student Activity Sheet 6 (one per student); graph paper (optional, one sheet per student)

Overview

Students find the point of intersection of two lines. Any strategy they have seen so far can be used.

About the Mathematics

Problems on this page are similar to problem 3. They are about finding the point of intersection of two lines. Only equations of the lines are presented and no graphs. A graphic as well as an algebraic method can be used to find the point of intersection. The algebraic method is more accurate and does not require students to make graphs first, which can be time-consuming.

Comments About the Solutions

4. and **5.**
Students may solve these problems using any strategy that they have learned in this unit. Encourage them to try and use an algebraic strategy at least once. If students have difficulty, you might remind them of the strategies they used to solve equations in previous sections.

6. This is a multi-step problem that integrates several concepts. Students need to find the point of intersection of the given lines. They then need to find the slope of the line from tower A to the point of intersection in order to write the equation of the line from A to the point of intersection.

9. It is likely that students find different answers for **b** and **c** since the coordinates are not whole numbers. Discuss that the algebraic method is more precise than the graphic method. If students want to use the graphic method for finding the point of intersection, they must check the answer using the equations.